UNIVERSITY OF Southampton
School of Social Sciences

Economics Division
University of Southampton
Southampton SO17 1BJ, UK

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Title: # Sorting Across Markets

By : Thomas Gall (University of Southampton),

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Sorting Across Markets*

Thomas Gall†

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Abstract

This paper examines the role of market intermediaries that provide trading parties with the institutional infrastructure that governs their contracting in determining market institutions and allocative efficiency in an economy. When there are limits to compensating partners, e.g. due to moral hazard problems and limited liability, and agents differ in their attributes, heterogeneity of market intermediaries in terms of contractual infrastructure may be needed to ensure that a competitive allocation attains a surplus efficient sorting of agents. This may involve the use of institutional infrastructure that is dominated in terms of joint surplus generated in any match, providing a rationale for a lack of convergence of institutional frameworks and the survival of outdated contractual institutions. A possible application is the use of certain types of contracts in venture capital markets. Competition of intermediaries by setting both institutional infrastructure and user fees can ensure an efficient degree of institutional diversity.
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JEL Codes: C78, D40, L10.

1 Introduction

Many economic interactions, in particular those between heterogenous parties, involve market intermediaries: financial intermediaries bring together investors and projects, (online) market places buyers and sellers, and media senders and receivers. Indeed, the role of intermediaries in facilitating

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†University of Southampton, Economics Division, School of Social Sciences, Southampton SO17 1BJ, UK; email: t.gall@soton.ac.uk

the meeting and matching of agents has been the subject of an active and growing literature.[1] What has received less attention, however, is the role of market intermediaries in providing institutional infrastructure for trading parties, in particular the contractual environment the parties have at their disposal. This could be in terms of verifiable information procured by auditing and monitoring, trustee services such as the holding of collateral, the design of bargaining procedures in case negotiations break down, or the legal system in a jurisdiction.

The nature of contractual institutions will be particularly relevant when economic interactions are complex, possibly involving problems of moral hazard between trading partners. This is true for instance in the supply of expert advice, in particular in health care, when financing entrepreneurs, and in procuring complex inputs. In such settings the fruits of economic interaction typically depend on the precise characteristics of those interacting. Hence, the assignment of agents will matter for both individual and aggregate payoffs. Yet contractual frictions, such as moral hazard and limited liability, will affect the sorting of individuals and may preclude a surplus efficient assignment, leading to mis-allocation (see Legros and Newman, 2007).[2]

This paper argues that despite the presence of contractual frictions institutional heterogeneity in form of market intermediaries that differ in their contractual institutions will ensure a surplus efficient sorting, which takes place across markets. This is because institutional heterogeneity facilitates the efficient sorting of agents, mitigating potential mis-allocation. Hence, a contractual institution that is inefficient if evaluated in isolation may still be desirable from a general equilibrium point of view. Finally, when market intermediaries compete first in the choice of institutional infrastructure and then in user fees, this induces sufficient heterogeneity of market intermediaries to ensure the surplus efficient sorting.

A simple formal setting to develop this argument is a model of expert advice, where agents on one market side demand counsel and expertise from those on the other side, for instance in investment banking, medical care, or legal services. Typically agents endowed with the most severe problems have greatest need of the most competent advice. This is modeled by let-

[1]See e.g. Rochet and Tirole (2006) for a survey on the literature on two-sided markets.

[2]For instance, introducing report cards determining physicians' payoffs in the market for certain types of heart surgery induced them to cherry-pick patients, e.g., using racial profiling (Werner et al., 2005), arguably a form of mis-allocation.

ting experts exert effort at a cost that depends on both the expert's quality and the severity of the agent's problem. This cost decreases in the expert's quality but more so for severe than for simple problems. Hence, to maximize aggregate surplus the best experts should treat the most severe problems.

Yet often precisely those agents that have more severe problems and conditions lack the means of compensation to attract expert advice: examples are entrepreneurs with little collateral, patients with complex conditions likely to constitute legal risks after treatment, or poor clients seeking damages from companies. Such settings tend also to be plagued by asymmetric information as effort is hard to observe and harder yet to verify. That is, a pair of principal and agent face a moral hazard problem under limited liability.

An example of the role of a market intermediary in providing institutional infrastructure, is the provision of a monitoring technology that yields an informative signal of the expert's effort. This allows parties to condition payments on the outcome of monitoring. The accuracy of an intermediary's monitoring then reflects contractual institutions by determining the feasible contracts parties have access to: higher accuracy of the monitoring technology increases aggregate surplus in a match and, by reducing the expert's information rent, changes its possible distribution.

Individuals have symmetric information about their types, that is, quality and severity; problems of asymmetric information arise only within a partnership of principal and agent. If an expert's cost facing a severe problem is large enough, then for any accuracy of monitoring in the competitive equilibrium of this market for advice the best experts pick the simplest problems, generating adverse sorting and mis-allocation of talent. The reason is that because of limited liability and borrowing constraints an agent with a severe problem cannot adequately compensate a good expert for the comparatively high effort cost. This immediately points to a possible remedy for the problem of adverse sorting: bringing in another intermediary with a less precise monitoring technology, which yields comparatively high information rents to experts. This allows high quality experts and principals with severe problems to use the new intermediary, while principals with simple problems and low quality experts remain with the original one.

To model endogenous choice of and competition in institutional infrastructure in the simplest conceivable manner let two intermediaries sequentially choose a sharing rule that determines the distribution of surplus in each match using that intermediary. Then they compete for matches by setting user fees in form of a fixed percentage of the surplus generated in a match. While this is best interpreted as jurisdictions competing in legislation and taxes, this also applies well to financial intermediaries, such as insurance brokers or underwriters. This setting is informative for the general case because, whenever the equilibrium outcome is a surplus maximizing matching involving the use of different intermediaries for different types of matches, this can be induced by heterogeneous intermediaries that are limited to offer a single surplus sharing rule. Applying standard methods to solve this game yields an analogy to Hotelling competition: the intermediaries choose sharing rules to maximize their distance. As the set of feasible sharing rules approaches $[0, 1]$ in the unique equilibrium of the game the surplus efficient sorting is achieved and both intermediaries obtain positive rents.

These arguments are readily applied to a particular type of relationship banking, the market for venture capital. Matching between entrepreneurs and venture capitalists in the U.S. is reportedly positive assortative in ability (Sørensen, 2007). Kaplan et al. (2007) find that experienced venture capitalists use a distinct contractual form (termed U.S. style), while less experienced venture capitalists use another (European style), sometimes incorporating in the appropriate jurisdiction if necessary.[3] U.S. style contracts are characterized by the use of many contingencies to determine future payoffs to the entrepreneur, not unlike a monitoring technology, thereby favoring venture capitalists. In contrast, European contracts are plain equity contracts, not making use of contingencies or milestones. Indeed, sorting across markets is consistent with these empirical observations, suggesting that European style contracts are not necessarily obsolete from a welfare point of view.

Related literature includes studies of frictionless matching markets when utility is not perfectly transferable. An early contribution is Legros and Newman (1996) where agents decide on the form of contract at the time of the match. Available types of contracts are exogenous and result in positive assortative matching of agents in wealth, which is inefficient from an aggregate surplus point of view.[4] Also some more recent papers study principal

[3] Also Lerner and Schoar (2005) document that choice of contract in the private equity industry tends to depend on a host country's legal institutions.

[4] Other reasons for why equilibrium matching might not be surplus-efficient include coordination failures, widespread externalities, and informational search frictions. In the case of the latter various remedies, typically with an aim to induce positive assortative matching, have been suggested (see e.g. McAfee, 2002, Jacquet and Tan, 2007). Positive

agent matching markets when asymmetric information within matches or renegotiation place bounds on feasible compensations, finding various types of surplus inefficiencies (See e.g. Chakraborty and Citanna, 2005, Dam and Perez-Castrillo, 2006, Gall, 2010, Legros and Newman, 2008, among others). Another study that emphasize sorting of heterogeneous agents into different contracts is Ghatak (1999), presenting a screening problem that can be solved using self-selection into groups under perfectly transferable utility. Besley and Ghatak (2005) studies a principal agent matching market with limits to compensation where the equilibrium outcome has positive assortative matching. In contrast to this study they focus on the case where this is also efficient. Common to this literature is that available types of contracts are exogenous and there is a single market place.

Some recent contributions in the literature on two-sided markets and platform competition focus on heterogeneity and sorting. In particular, coexistence of multiple platforms in a competitive equilibrium has been found as a result of congestion effects (Ellison et al., 2007), or because platforms specialize in attracting particular types of agents who differ in their valuations (Ambrus and Argenziano, 2009) or in their attractiveness (Damiano and Li, 2008). This strand of literature usually abstracts from explicitly modeling matching within platforms or the choice of contracting environments, leaving no room for specialization of platforms in the types of contracts available to partners in match.

Competition in institutional settings has received less attention. One instance is Bierbrauer and Boyer (2010) who study political competition in nonlinear income taxation regimes. Another one, Nocke et al. (2007), compares outcomes on a market platform depending on whether residual control rights of platform access reside with agents from a market side or with a third party, they do not consider the presence of multiple heterogenous platforms.

The paper is organized as follows. Section 2 presents a basic framework to illustrate sorting across markets and considers an application to the venture capital market. Section 3.2 models competition of market places in institutional arrangements, while Section 4 concludes.

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assortative matching has been documented empirically for a number of markets, however, suggesting that informational search frictions can be overcome. For instance, Chen (2008) finds that in the U.S. larger banks tend to fund larger projects. Fernando et al. (2005) report that higher ability underwriters tend to associate with higher quality equity issuers.

2 A Market for Advice

The essence of the argument in this paper can be demonstrated in a simple market for medical, legal, or economic advice, where a continuum of expert advisors A trades with a continuum of principals B. Let $\mu \in (0,1)$ denote the measure of advisors and $1-\mu$ the measure of principals. Suppose all principals have zero wealth at the time of forming business relationships and no access to credit, but they can contract on future income.[5] Advisors are protected by limited liability.

2.1 Technology

A typical principal faces an uncertain income, which is either 1 in case of success, with probability p, or 0 otherwise. A principal is characterized by a type $b \in \{\ell; h\}$. b reflects the ease of success of the principal's case. Principals have linear utility in income. The probability of success $p(e)$ can be increased by an advisor who can exert effort e on the principal's behalf. This describes well situations where individuals have an opportunity for future revenue that benefits from expert advice, but are cash constrained at least at the time of forming the business relationship, for instance litigation, medical consulting, loan brokerage, or entrepreneurship.

Let $e \in \{0; 1\}$ and $p(e) = e$. Advisors have linear utility in income and incur a utility cost of $e^2/(2r(a,b))$ when choosing effort e. $r(a_i, b_i)$ determines the advisor's cost of effort in a match (i,j) with attributes a_i and b_j. Unmatched advisors have utility 0, as do unmatched principals. Denote the singleton payoff by $\underline{u}_i(h) = \underline{u}_i(\ell) = 0$ for $i = a, b$. Suppose that

$$0 < r(\ell,\ell) < r(\ell,h) = r(h,\ell) < r(h,h),$$

that is, the easier the case and the more able the advisor the lower the marginal cost of effort.

2.2 Intermediary

Principal and advisors meet and match into pairs in a market. An intermediary provides an institutional framework that determines which types of

[5] This can be relaxed; ex ante wealth is required to be sufficiently small for some principals only, which seems plausible in the context of entrepreneurship, healthcare, or litigation versus companies.

contracts a pair of principal and advisor who wish to enter a relationship can use. Here, the intermediary enforces a feasible payment w from the principal to the advisor contingent on verifiable information and provides a monitoring device that sends a verifiable signal on the effort the advisor has chosen. The device detects whether the advisor's effort does not match a pre-specified effort level e_0, but is correct only with a probability $q > 1/2$. The benchmark effort level e_0 is not contractible at the matching stage; this is typically the case if details of the project can only be described meaningfully in the process of collaboration. The intermediary does not face any cost. Here the intermediary may be interpreted as a financial firm taking deposits and offering auditing services in the spirit of Diamond (1984), as an insurance company paying and auditing the advisor to remedy an insured damage to the principal, or as a legal system that detects breach of contract only imperfectly.

2.3 Contracts

Contracts at the matching stage thus specify wages possibly depending on success or failure of the project, and on the monitoring outcome. Because of the principals' wealth constraints only a payment of 0 is feasible in case of failure.[6] In fact, a contract only specifies a wage $w \in [0, 1]$ that the principal pays to the advisor in case of success. The principal is free to renege on that payment if the monitoring device detects that the advisor did not choose the pre-specified effort.[7]

2.4 Timing

To sum up the timing is as follows.

1. Nature chooses types a and b.
2. Pairs of principals and advisors meet in a market and match in pairs agreeing on a wage w.
3. Principals specify target effort levels e_0.

[6] Sufficient conditions for the argument to go through are either sufficiently small wealth of all principals, or that a single failure wage is imposed on all pairs of principals and advisors, such as a base wage for the expert paid by an insurance.

[7] Allowing for two different success wages that depend on the result of monitoring does not change the results meaningfully, but reduces clarity of exposition considerably.

4. Advisors choose effort levels e.

5. Success or failure realizes, monitoring device reports, and payoffs accrue accordingly.

2.5 Feasible Contracts

Given e_0, an advisor who chooses e has expected payoff

$$u^a = e_0 q w - \frac{e^2}{2r(a,b)} \text{ if } e = e_0 \text{ and } u^a = e(1-q)w - \frac{e}{2r(a,b)} \text{ otherwise.}$$

This means the highest effort that can be implemented with monitoring accuracy q is

$$e_0 \leq \left(q + \sqrt{2q-1}\right) wr(a,b).$$

Once e_0 can be meaningfully described, it is set by the principal who finds it always optimal to set e_0 as high as possible. This yields payoffs

$$v^a = (1-q)^2 \frac{r(a,b)w^2}{2} \text{ and } v^b = (q + \sqrt{2q-1})(1-qw)r(a,b)w. \quad (1)$$

Note that a principal's maximal payoff is associated to wage $1/(2q)$ and given by $(1 + \sqrt{2q-1}/q)r(a,b)/4$. An advisor's payoff is maximal for $w = 1$ (since limited liability requires that $w \leq 1$), yielding $(1-q)^2 r(a,b)/2$. Hence, an intermediary that offers monitoring accuracy q gives a match (a,b) access to payoffs

$$\phi(a,b) = \left\{(u^a, u^b) \in \mathbb{R}_0^+ \times \mathbb{R}_0^+ : u^a \leq v^a(a,b,w), u^b \leq v^b(a,b,w), w \in \left[\frac{1}{2q}, 1\right]\right\}.$$

That is, q fully determines all attainable payoff combinations, denoted by $\phi(a,b)$, that can be agreed upon by a match (a,b) when choosing a match using the intermediary, i.e. the Pareto or utility possibility frontier. Figure 1 depicts the set of attainable surplus distributions for $q \geq 2 - \sqrt{q}$ implying that first best joint surplus $r(a,b)/2$ can be reached, which is represented by the thin lines. That is, q is best interpreted as contractual or market institutions underlying transactions in the market for advice.

The market equilibrium is given by a stable assignment of advisors to principals given the contractual institutions of the intermediary, i.e. attainable payoffs $\phi_\alpha(a,b)$, $a,b = \ell, h,$.

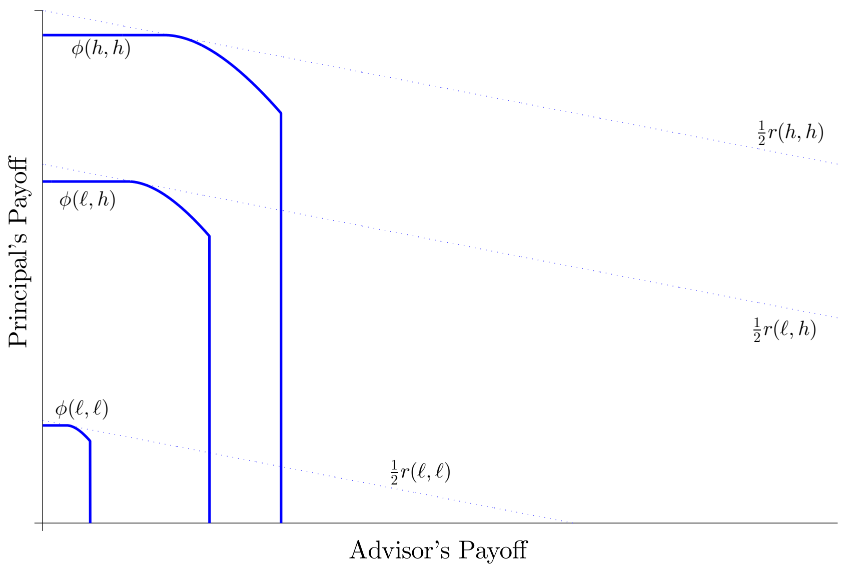

Figure 1: Attainable surplus distributions with a monitoring intermediary.

Definition 1 (Market Equilibrium) *An market equilibrium allocation with an intermediary providing attainable payoffs ϕ specifies a measure-consistent partition of the agent space $A \cup B$ into pairs (i, j) or singletons (i) or (j) of agents $i \in A$ and $j \in B$, and payoffs $(u_i)_{i \in A \cup B}$ such that*

- *payoffs in any equilibrium match (i, j) are feasible with respect to ϕ, i.e. $(u_i, u_j) \in \phi(a_i, b_j)$, unmatched agents obtain $\underline{u}_a(a)$ and $\underline{u}_b(b)$ and*
- *(i) all agents obtain at least the payoff they have when unmatched, and (ii) there is no pair $i \in A$ and $j \in B$ who are not matched in the equilibrium allocation, but both strictly prefer some $(u'_i, u'_j) \in \phi(a_i, b_j)$ to their equilibrium payoff.*

Measure consistency is needed to ensure that the measure of matched A agents equals the one of B agents. Existence of such a market equilibrium allocation is straightforward and follows for instance from Kaneko and Wooders (1986), which also discusses measure consistency.

The market equilibrium allocation will depend on whether low types on both market sides find it profitable to outbid high types on their market side for high types on the other side, and on whether such outbidding is feasible given the market institutions, that is whether the associated payoffs lie in ϕ. Turning to the former, low types have higher additional valuation for high types than have high types if

$$r(h,\ell) - r(\ell,\ell) > r(h,h) - r(\ell,h). \tag{DD}$$

This will typically be the case if higher quality of an advisor matters more for complex cases. To determine the equilibrium matching pattern a result in Legros and Newman (2007) is helpful: positive assortative matching (as much (h,h) pairs as possible) is implied by the property of generalized increasing differences. The following proposition uses a sufficient condition for generalized increasing differences to characterize the equilibrium matching pattern, its proof can be found in the Appendix.

Proposition 1 *Suppose that* $q \in [1/2, 1]$ *and*

$$\sqrt{r(h,h)/r(h,\ell)} > 5/4. \tag{GID}$$

Then a market equilibrium with an intermediary providing payoffs ϕ *exhausts all potential* (h,h) *matches, assigns remaining* h *agents to* ℓ *agents, and exhausts all remaining possible* (ℓ,ℓ) *matches. Otherwise, an equilibrium in a market place with payoffs* ϕ *exhausts all potential* (h,ℓ) *and* (ℓ,h) *matches, and matches all remaining agents into homogenous* (h,h) *or* (ℓ,ℓ) *matches.*

Note that the conditions (DD) and (GID) are not mutually exclusive. That is, an equilibrium in market place may not generate the efficient matching pattern whenever the ratio $r(h,h)/r(h,\ell)$ is sufficiently great. Moreover, both conditions do not depend on the monitoring accuracy q. This implies the following corollary:

Corollary 1 *Suppose conditions (DD) and (GID) hold. Then for any monitoring accuracy* $q \in [1/2, 1]$ *there will be mismatch in a market equilibrium, i.e. the matching pattern will not maximize aggregate surplus.*

2.6 Two intermediaries

Proposition 1 established that the payoffs attainable with an intermediary providing monitoring accuracy $q \in [1/2, 1]$ may not permit sufficient flexibility of compensation within a match to enable the surplus efficient sorting in equilibrium. The question arises whether sufficient institutional diversity may mitigate such inefficiencies.

To address this issue suppose now that two intermediaries provide different monitoring accuracies, and agents choose which intermediary to use.

Denote the intermediaries by 1 and 2 offering monitoring quality q_1 and q_2 respectively. Assume that a principal and an advisor choose the intermediary for their contracting at the time of matching. Denote this choice by $c_i, c_j \in \{1; 2\}$, and necessarily $c_i = c_j$ if i and j are matched. Unmatched agents use neither intermediary 1 nor 2. This corresponds to multi-homing in the two-sided markets literature as the intermediary can be chosen at the time of contracting.

Formally, an intermediary m is characterized by the set of attainable payoffs contingent on a match (a, b), $\phi_m(a, b) : \{\ell; h\} \times \{\ell; h\} \rightrightarrows \mathbb{R}^2$, it gives access to. An equilibrium with multiple intermediaries is defined as follows.

Definition 2 (Equilibrium Across Markets) *A market equilibrium allocation with intermediaries* $\{m_1, m_2, ..., m_n\} =: M$ *with attainable payoffs* $(\phi_m)_{m \in M}$ *specifies a measure-consistent partition of the agent space* $A \cup B$ *into pairs* (i, j) *or singletons* (i) *or* (j) *of agents* $i \in A$ *and* $j \in B$*, location choices* $(c_i)_{i \in A \cup B}$*, and payoffs* $(u_i)_{i \in A \cup B}$ *such that*

- *unmatched agents choose* $c_i = \emptyset$ *and obtain* $\underline{u}_a(a_i)$ *and* $\underline{u}_b(b_j)$*,*
- *matched agents* (i, j) *choose* $c_i = c_j$ *with* $c_i, c_j \in M$*,*
- *for all* $m \in M$ *the set of matches* (i, j) *with* $c_i = c_j = m$ *and associated payoffs* (u_i, u_j) *are a market equilibrium with intermediary* m *for all* $i \in A$ *and* $j \in B$ *with* $c_i, c_j = m$*,*
- *there is no pair of agents* $i \in A$ *and* $j \in B$ *and an intermediary* m *such that* (i, j) *are not matched with* $c_i = c_j = m$ *in equilibrium, but both strictly prefer some* $(u_i', u_j') \in \phi_m$ *to their equilibrium payoffs.*

That is, when evaluating a possible match agents use the Pareto optimal intermediary. Attainable payoffs with multiple intermediary are an aggregate of the sets of attainable payoffs for the individual intermediaries. Since agents can choose the intermediary at the time of contracting, the set of attainable payoffs across a set of intermediaries $M = \{m_1, ..., m_n\}$ is simply the union of the sets of attainable payoffs in the individual intermediaries, $\bigcup_{m \in M} \phi_m(a, b)$ for $a, b = \ell, h$. Note that both equilibrium concepts, with one or multiple intermediaries, require stable assignments, postulating that equilibrium payoffs are individually rational and stable with respect to deviations of pairs (a, b) using attainable payoffs for that intermediary, respectively for all intermediaries. Therefore an equilibrium with intermediaries M is equivalent to an equilibrium with one intermediary providing attainable payoffs $\bigcup_{m\in M}\phi_m$ as is stated in the following proposition; its proof is in the appendix.

Proposition 2 *A measure consistent partition of the agent space $A\cup B$ into pairs (i,j) with $c_i = c_j \in \{m_1, m_2, ..., m_n\} =: M$ and singletons (i) and (j) with $c_i = \emptyset = c_j$, and payoffs $(u_i)_{i\in I}$ is an equilibrium with intermediaries M with attainable payoffs $(\phi_m)_{m\in M}$ if, and only if, the assignment of agents and payoffs are an equilibrium with an intermediary with attainable payoffs $\phi(a,b) = \bigcup_{m\in M}\phi_m(a,b)$ for $a = \ell, h$ and $b = \ell, h$.*

Hence, multi-homing can be understood as a means to aggregate contractual structures of many intermediaries in an economy; it remains an equilibrium with rational expectations under single-homing. To solve for an equilibrium when two intermediaries 1 and 2 that differ in monitoring quality q, set $q_1 = 1/2$ in order to considerably simplify exposition. That is, intermediary 1's monitoring does not provide an informative signal. Using (1), attainable payoffs using intermediary 1, ϕ_1 are given by

$$\phi_1(a,b) = \left\{(u^a, u^b) \in \mathbb{R}_0^+ \times \mathbb{R}_0^+ : u^a \le r(a,b)/8, u^b \le r(a,b)/4\right\},$$

for $(a,b) \in \{\ell; h\}^2$. That is, $q = 1/2$ and limited liability ($w \le 1$) generate strictly non-transferable utility for intermediary 1. Assume that $1/2 < q_2 \le 1$ then attainable payoffs in market 2 are given by

$$\phi_2(a,b) = \left\{(u^a, u^b) \in \mathbb{R}_0^+ \times \mathbb{R}_0^+ : u^a \le v^a(a,b,w), u^b \le v^b(a,b,w), w \in \left[\frac{1}{2q_2}, 1\right]\right\},$$

with $v^a(.)$ and $v^b(.)$ defined in (1). Note that as the monitoring quality q increases, so does the maximal payoff of the principal. Likewise, the advisor's maximal payoff decreases in q. That is, a likely allocation to sustain (h,ℓ) and (ℓ,h) and discourage (h,h) matches, will have (h,ℓ) matches in market 1 and (ℓ,h) matches in market 2. Hence, to ensure that this is stable and thus a market place equilibrium in $\phi_1(a,b) \cup \phi_2(a,b)$

$$\frac{r(h,\ell)}{8} > \frac{r(h,h)}{2}(1-q_2)^2, \tag{IC A}$$

which is the highest payoff an h advisor can obtain in a (h,h) match with intermediary 2. Likewise, an h principal's payoff in a (h,h) match with intermediary 1 has to be less than the payoff in a (ℓ,h) match with intermediary 2, which is bounded below by

$$(q_2 + \sqrt{2q_2 - 1})(1-q_2)r(h,\ell) > \frac{r(h,h)}{4},. \tag{IC B}$$

Closer inspection reveals that incentive compatibility for the advisor binds, and thus for any monitoring quality in market 2

$$q_2 > 1 - \frac{1}{2}\sqrt{\frac{r(h,\ell)}{r(h,h)}} := \underline{q}$$

all h agents strictly prefer a (h, ℓ) match using intermediary 1 ((ℓ, h) using 2) to matching with a h agent from the other market using intermediary 2 (1). By (DD) $r(h,\ell)/r(h,h) > 1/2$. Note also that all ℓ principals strictly prefer a (h, ℓ) match using intermediary 1 to a (ℓ, ℓ) match using any intermediary if

$$\frac{r(h,\ell)}{4} > (q + \sqrt{2q_2 - 1})\frac{r(\ell,\ell)}{4q},$$

which is equivalent to $r(h,\ell)/r(\ell,\ell) > 1 + \sqrt{2q-1}/q$. This must hold for all $q_2 \in [1/2, 1]$, since under (DD) and (GID) $r(h,\ell)/r(\ell,\ell) > 2$. That is, h agents from A (B) prefer heterogenous matches with intermediary 1 (2), and ℓ principals prefer a heterogenous match with intermediary 1. Hence, in equilibrium all possible (h, ℓ) matches are exhausted, any remaining h advisors match with h principals. If $\mu \geq 1 - \mu$ advisors are abundant any remaining h principals therefore match with ℓ advisors with intermediary 2, since this yields strictly higher payoff to ℓ advisors than staying solitary. If $\mu < 1 - \mu$ ℓ surplus h and ℓ principals compete for ℓ advisors. There is a feasible contract provided by intermediary 2 such that ℓ advisors prefer a (ℓ, h) match in 2 to a (ℓ, ℓ) match in 1 if

$$\frac{r(\ell,\ell)}{8} < \frac{r(\ell,h)}{2}(1 - q_2)^2,$$

where the left hand side gives an upper bound of an ℓ advisor's payoff in a (ℓ, h) match. This condition is equivalent to

$$q_2 < 1 - \frac{1}{2}\sqrt{\frac{r(\ell,\ell)}{r(h,\ell)}} := \overline{q}$$

Since by (DD) $r(h,\ell)/r(h,h) > r(\ell,\ell)/r(h,\ell)$, $\overline{q} > \underline{q}$.

That is, the presence of heterogenous intermediaries generates different sub-markets that allows the efficient sorting to occur across markets, which is stated in the following proposition.

Proposition 3 (Sorting Across Markets) *Suppose both (DD) and (GID) hold. Consider intermediaries* 1 *and* 2*, characterized by monitoring quality*

$q_1 = 1/2$ and q_2 that determine attainable payoffs ϕ_1 and ϕ_2 as above. Then there are $1/2 < \underline{q} < \overline{q} \leq 1$ such that for $q_2 \in [\underline{q}, \overline{q}]$ sorting across market occurs: all possible (h, ℓ) matches form using intermediary 1, all possible (ℓ, h) matches form using intermediary 2, and the remaining agents exhaust all possible homogeneous matches with intermediary 1 (2) when $\mu > 1 - \mu$ $(\mu < 1 - \mu)$. If principals are scarce $(\mu \geq 1 - \mu)$ $\overline{q} = 1$.

Figure 2 gives a graphical illustration of this finding. Dashed and solid lines correspond to the utility possibility frontiers offered by intermediaries 1 and 2. The set of attainable payoffs with both intermediaries is simply the union of the areas under the two utility possibility frontiers for each combination (a, b). Note that there are now payoffs in the union of the utility possibility frontiers such that h agents prefer heterogeneous matches to (h, h) matches using the other intermediary, and ℓ agents prefer heterogeneous matches to (ℓ, ℓ) matches using any intermediary.

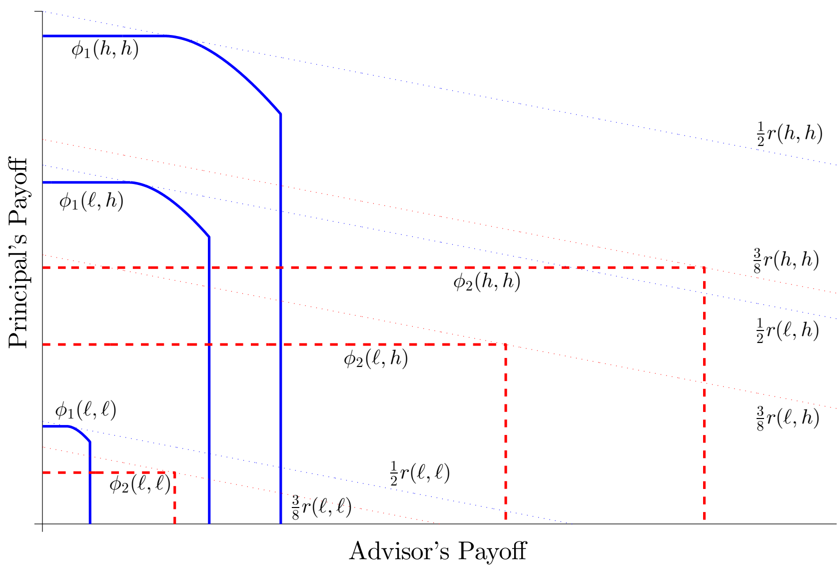

Figure 2: Attainable surplus distributions on markets 1 and 2 with $q_1 = 1/2$ and $q_2 > q_1$.

In fact, the joint surplus for any match (a, b) is strictly lower when using intermediary 1 than with intermediary 2. That is, despite the fact that one intermediary strictly dominates the other one in terms of joint surplus, there are constellations such that both intermediaries attract agents who match, and this increases aggregate surplus compared to an equilibrium with the dominant intermediary 2 only. Proposition 3 therefore highlights a potential trade-off in the choice of contractual institutions: between maximizing aggregate surplus given the match in a market and possible benefits of mitigating mismatch through sorting across markets. This gives a rationale for the sustainability of market institutions that, by themselves, do not appear to be surplus efficient. Finally, note that neither does (i) imply the condition in (ii) nor vice versa.

2.7 Application: Market for Venture Capital

One possible application of the model outlined above is the market for venture capital where entrepreneurs are matched to venture capitalists. Entrepreneurs differ in the quality of their projects and exert effort that affects the probability of success. Sørensen (2007) presents evidence from the U.S. that the success of a start up is positively affected by the experience of the venture capitalist, even when controlling for the fact that more experienced venture capitalists tend to pick better entrepreneurs. This is despite the fact that given the econometric model used for estimation, positive assortative matching (i.e. better entrepreneurs obtain better financiers) is inefficient. Hsu (2004) documents that experience commands a market price in terms of higher cash flow rights for more experienced venture capitalists.

As for contractual institutional institutions Kaplan and Strömberg (2003) document that contracts between entrepreneurs and venture capitalists tend to be of one of two following forms: (i) simple equity contracts that assign fixed shares of the profit to the parties at the beginning of the relationship, which they call European style contracts, and (ii) complex option contracts that make the distribution of profit and control rights in the venture dependent on verifiable events, called U.S. style contracts, based on the geographic prevalence of the types.

Finally, there is some evidence that venture capital contracts depend on the jurisdiction used for contracting and that parties to a trade choose the jurisdiction appropriately: Lerner and Schoar (2005) report that the quality of the legal system affects the choice of contractual form in the private equity industry, and Kaplan et al. (2007) find that the location of incorporation is sometimes used to select into different contractual forms. The latter also provide evidence on the type of contracts employed in venture capital deals all over the world. They report that more experienced venture capitalists use contracts that generate high-powered incentives for the agent (U.S. style contracts) whereas less experienced venture capitalists use lower powered incentives. Venture capitalists in their sample tend to use high-powered incentives more as they become more experienced; those who use low-powered incentives tend to be less successful. While this seems consistent with financiers learning about the best contractual form over time, the same observation can be generated by a sorting equilibrium of venture capitalists across markets, as will be demonstrated.

In the model context U.S. style contracts seem well represented by the use of an informative monitoring device that sends a signal about the entrepreneurs' effort choice. Specifying e_0 then amounts to choosing the milestones that the entrepreneur needs to satisfy to not lose control and cash flow rights; given that financing relations are often characterized by a high degree of lock-in of the entrepreneur the possibility of the financier to choose e_0 ex post seems plausible. Assuming limited liability of venture capital financiers seems less plausible, however. Let us therefore dispense with the assumption and allow for wages $w > 1$. This means attainable payoffs under U.S. style contracts are given by

$$\phi_{US}(a,b) = \left\{(u^a, u^b) \in \mathbb{R}_0^+ \times \mathbb{R}_0^+ : u^a \leq v^a(a,b,w), u^b \leq v^b(a,b,w), w \in \left[\frac{1}{2q}, \frac{1}{q}\right]\right\}.$$

On the other hand, a European contract corresponds to agreeing on a fixed sharing of the joint profit. An example is the case of $q = 1/2$ above, inducing an equal split of the surplus. More generally, suppose that monitoring is not used or uninformative, and let $s \in [0,1]$ denote the share of surplus that accrues to the entrepreneur. The entrepreneur then optimally chooses effort

$$e = s/r(a,b).$$

This yields payoffs depending on the sharing rule s:

$$v_e^a = s^2 r(a,b)/2 \text{ and } v_e^b = s(1-s)r(a,b).$$

Note that $s = 1$ maximizes the joint surplus. Hence, attainable payoffs under European style contracts are given by

$$\phi_{EU}(a,b) = \left\{(u^a, u^b) \in \mathbb{R}_0^+ \times \mathbb{R}_0^+ : u^a \leq v_e^a(a,b,s), u^b \leq v_e^b(a,b,s), s \in \left[\frac{1}{2}, 1\right]\right\}.$$

Figure 3 shows the sets of attainable payoffs. Notice that attainable payoffs induced by a sharing rule s coincide with the ones induced by a monitoring contract with accuracy $q = 1/2$ and wage $w = 2s$, so that this setting is indeed a simple extension of the basic model dispensing with limited limited on the side of the principals.

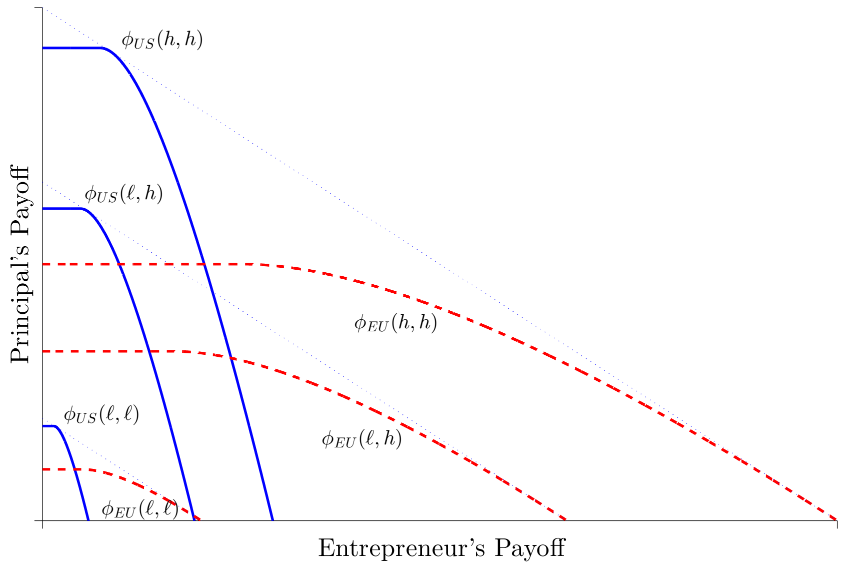

Figure 3: Attainable payoffs under U.S. and European style contracts.

Indeed our results from above go through almost without qualification when capital is scarce, that is $\mu < 1/2$. If entrepreneurs are abundant, some ℓ type entrepreneurs will remain unmatched. Therefore any ℓ entrepreneur in a match (ℓ, b) will obtain the minimum attainable payoff in that match, for instance $r(\ell, b)/8$ if only European contracts are available. This pins down the principals' payoffs when matching with ℓ entrepreneurs, which in turn determines h type entrepreneurs' payoffs in (h, ℓ) matches. Computing these payoffs to derive the equilibrium allocations in the three possible regimes yields the following proposition, details are in the appendix.

Proposition 4 *Suppose that capital is scarce and $r(a, b)$ satisfies (DD). If* $\sqrt{r(h,h)} + \sqrt{r(h,h) - r(h,\ell)} > \sqrt{r(h,\ell)} + \sqrt{r(h,\ell) - r(\ell,\ell)}$ *then*

(i) when either only European or only U.S. style contracts are available all possible (h, h) matches are exhausted,

(i) when both European and U.S. style contracts are available all possible (h, ℓ) and (ℓ, h) matches are exhausted, and all h financiers and all ℓ entrepreneurs use U.S. style contracts.

Hence, when $r(a, b)$ has decreasing differences, but mildly so, matching in the market for venture capital with only European or only U.S. style contracts is strictly positive assortative, although heterogeneous matches would yield higher aggregate surplus. When both contract types are available there occurs sorting across market, which increases aggregate surplus compared to both contracts in isolation.

But note that there is still monotone matching conditional on the type of contract used: ℓ principals using U.S. style contracts are matched only with ℓ entrepreneurs, if any, whereas h principals using U.S. style contracts are matched to ℓ entrepreneurs but also to h entrepreneurs should any h principals remain. Hence, the observation of more experienced financiers using a particular kind of contract and monotone matching of more experienced financiers to better entrepreneurs can be explained by a sorting argument. In this case a move to homogenize contractual institutions, rendering European style contracts more American by facilitating the enforcement of option clauses that transfer control right in pre-specified events, appears not advisable from a welfare point of view.

3 Competition in Contractual Institutions

Given the observations above it is of interest how heterogeneity in contractual institutions may emerge endogenously, and whether it is sufficient to generate efficient sorting. Modeling competition of intermediaries in arbitrary contractual institutions poses a complex problem; to simplify it somewhat consider contractual institutions that generate linear sharing rules. As far as one is interested in the sorting this is without loss of generality: whenever, given an intermediary m, there exist another intermediary with attainable payoffs that do not Pareto dominate attainable payoffs of m and induce sorting across markets, this can be done using only linear surplus sharing rules. Stating this formally requires some notation. Say that sorting across markets occurs whenever an equilibrium with intermediaries m and m' exhausts all possible (h, ℓ) using one intermediary and all possible (ℓ, h) matches using the other one.

Proposition 5 *Suppose m is an intermediary with attainable payoffs $\phi^m(a, b)$. If there is an intermediary m' with ϕ', such that an equilibrium with intermediaries m and m' induces sorting across markets, then there are $y(a, b) =$*

$u_a' + u_b'$ *with* $(u_a', u_b') \in \phi'(a,b)$ *and* $\alpha \in [0,1]$ *with* $\phi_\alpha(a,b) = \{(u_a, u_b) : u_a \leq \alpha y(a,b), u_b \leq (1-\alpha) y(a,b)\}$*, such that an equilibrium with intermediaries* m *and* α *induces sorting across markets* m *and* α*.*

The proof is in the appendix, though its argument is straightforward: all that is needed to attract (h,ℓ) matches is to enable both agents an adequate payoff, that is, the condition for sorting across markets is on the attainable payoff for (h,ℓ) matches when using intermediary m' only. Since joint surplus is at most $y(h,\ell)$ for m', all possible payoffs for (h,ℓ) matches can be replicated using a linear sharing rule.

3.1 Linear Sharing Rules

Suppose that only linear surplus sharing rules are used and aggregate surplus does not depend on the sharing rule used. Denote joint surplus of partners in a match (a,b) by $y(a,b) = r(a,b)/2$. An intermediary m that offers a sharing rule s_m, specifying the surplus share accruing to the partner from market side A, gives access to attainable payoffs

$$\phi_m(a,b) = \{(u_a, u_b) : u_a \leq s_m y(a,b), u_b \leq (1-s_m) y(a,b)\}. \tag{2}$$

For instance $s_m = 1/2$ corresponds to the dashed lines in Figure 2. The following proposition characterizes the equilibrium allocation depending on the measure of principals μ in an economy with heterogeneous intermediaries 1 and 2 that induce sorting across markets; its proof is in the appendix.

Proposition 6 *Let property (DD) hold and intermediaries* 1 *use a surplus sharing rule* $s_1 \in [0,1]$ *with attainable payoffs* $\phi_1(a,b)$*. If*

$$\mu > 1-\mu \text{ and } s_1 \leq \frac{y(h,h)}{y(h,\ell)} \frac{y(h,\ell) - y(\ell,\ell)}{y(h,h) - y(\ell,\ell)}, \text{ or}$$
$$\mu < 1-\mu \text{ and } s_1 \geq \frac{y(\ell,\ell)}{y(h,\ell)} \frac{y(h,h) - y(\ell,h)}{y(h,h) - y(\ell,\ell)},$$

then there is $s_2 \in [0,1]$ *with attainable payoffs* $\phi_2(a,b)$ *such that in an equilibrium with intermediaries* 1 *and* 2 *(i) a positive measure of agents match in each market place, (ii) all potential* (h,ℓ) *and* $\ell, h)$ *matches are exhausted, and (iii) aggregate surplus is maximized.*

That is, Proposition 3 extends in that for sufficiently low $y(\ell,\ell)$ sorting across markets is possible for all $s_1 \in [0,1]$. More intriguingly, Proposition 6 states that, depending on $y(a,b)$ there may sharing rules s_1 such that there does not exist s_2 so that both platforms attract matches and aggregate is maximized. This is relevant if intermediaries compete in the institutional setup determining attainable payoffs ϕ: an incumbent might then choose α in order to prevent an entrant from choosing a β that attracts some matches and induces efficient sorting across market.

3.2 Competing Intermediaries

Suppose now that that two intermediaries, 1 and 2, compete for customers. Each intermediary m is characterized by its contractual institution, that is, a sharing rule s_m giving partners in a match (a,b) access to attainable payoffs $\phi_m(a,b)$ as defined in (2). The intermediary also charges a percentage fee $\tau_i \in [0,1]$ of the joint surplus $y(a,b)$ of a match (a,b). $y(a,b)$ can be interpreted as profit in a partnership, and τ_m as a profit tax and as commission fee.[8] Suppose for technical reasons that the set of feasible sharing rules is constrained and s_m cannot take extreme values, that is for $m = 1, 2$

$$s_m \in [\epsilon, 1-\epsilon],$$

where $\epsilon \in (0, 1/2)$ is small but positive. The analysis below will be interested in the case of ϵ approaching 0, hence this assumption is indeed rather innocuous.[9]

Agents on both market sides choose an intermediary for contracting given contractual institutions and fees. The solution concept for agents' choices is an equilibrium with multiple intermediaries. The timing is as follows.

1. An incumbent chooses contractual institutions, i.e., a sharing rule s_1.

2. An entrant chooses contractual institutions, i.e., a sharing rule s_2.

3. Given s_1 and s_2 intermediaries simultaneously set fees τ_1 and τ_2.

[8] Extending the model to allow for a commission levied on turnover rather than surplus is straightforward. This introduces a set of participation constraints for agents on τ_m that considerably complicates the analysis. The main result carries over, however.

[9] Allowing for, say, $s_1 = 0$ and $s_2 = 0$, induces the possibility of multiple equilibria when agents choose matches and intermediaries. In particular, when $\tau_1 = \tau_2 = 1$ any behavior of the agents is stable and thus an equilibrium with intermediaries 1 and 2. Letting ϵ approach 0 selects one of these equilibria that has the virtue of ensuring continuity of the outcome in terms of payoffs and allocation.

4. Given contractual institutions s_1 and s_2 and fees τ_1 and τ_2, agents match using intermediaries 1 or 2.

Intermediary m's profit is $\tau_i y(a, b)$ for each match (a, b) that uses this intermediary. That is, all sharing rules $s_m \in [0, 1]$ induce the same cost of 0.[10] Let market side A outnumber market side B, $\mu > 1 - \mu$. Denote the share of h types on market side A by p and the one on market side B by q.

Proceed backwards in time and start with the equilibrium with intermediaries 1 and 2. Refer to the sharing rule that gives more rent to market side A by α, and the other one by β, that is $\alpha > \beta$. The case $\alpha = \beta$ is treated below. Suppose intermediary α sets fee τ_α and β sets τ_β. Then three different cases may occur.

Lemma 1 *Given α and β with $\alpha > \beta$ and fees τ_α and τ_β, three cases may occur*

(i) intermediary α captures the entire market if

$$\tau_\beta > 1 - (1 - \tau_\alpha)\frac{1 - \alpha}{1 - \beta} := \tau_\beta^S(\tau_\alpha), \tag{3}$$

(ii) intermediary β captures the entire market if $\tau_\beta \leq \tau_\beta^M(\tau_\alpha)$, where

$$\tau_\beta^M(\tau_\alpha) = 1 - (1 - \tau_\alpha) \begin{cases} \frac{1-\alpha}{1-\beta} & \text{if } \frac{1-\beta}{1-\alpha} \leq \frac{\beta}{\alpha}\frac{y(h,h)}{y(h,\ell)} \\ \frac{1-\alpha}{1-\beta}\frac{y(h,\ell)}{y(\ell,\ell)} & \text{if } \frac{\beta}{\alpha}\frac{y(h,h)}{y(h,\ell)} < \frac{1-\beta}{1-\alpha} < \frac{\beta}{\alpha}\frac{y(h,h)}{y(\ell,\ell)} \\ \frac{\alpha}{\beta}\frac{y(h,\ell)}{y(h,h)} & \text{otherwise,} \end{cases}$$

with a strict inequality whenever $\tau_\beta^S(\tau_\alpha) > \tau_\beta^M(\tau_\alpha)$.

(iii) intermediary α attracts all (h, ℓ) matches, while β attracts all other matches if $\tau_\beta^S(\tau_\alpha) > \tau_\beta^M(\tau_\alpha)$ and

$$\tau_\beta^S(\tau_\alpha) \geq \tau_\beta \geq \tau_\beta^M(\tau_\alpha) \text{ and } \tau_\beta^S(\tau_\alpha) \geq \tau_\beta \geq \tau_\beta^M(\tau_\alpha).$$

Denote the aggregate surplus in matches that use intermediary m by y^M if m attracts all matches, and by y_m^S if α attracts (h, ℓ) and β all other matches;

[10] One could make one intermediary less cost efficient, for instance by requiring $\tau \geq \underline{\tau}$ for some constant $\underline{\tau}$ for that intermediary.

they are given by

$$\begin{aligned} y^M =&(1-q)(1-\mu)y(\ell,\ell)+\min\{p\mu;q(1-\mu)\}[y(h,h)-y(\ell,\ell)] \\ &+\max\{p\mu;q(1-\mu)\}[y(h,\ell)-y(\ell,\ell)], \\ y^S_\alpha =&\min\{p\mu;(1-q)(1-\mu)\}y(h,\ell), \text{ and} \\ y^S_\beta =&\min\{p(1-\mu);q(1-\mu)\}y(h,\ell)+\max\{p(1-\mu)-q(1-\mu);0\}y(h,h) \\ &+\max\{q(1-\mu)-p(1-\mu);0\}y(\ell,\ell). \end{aligned}$$

If the other intermediary attracts all matches, aggregate surplus of matches using m is 0. Note that $2y^S_\beta > y^M > y^S_m$ for $m=\alpha,\beta$, but both $y^S_\alpha > y^S_\beta$ and the reverse may occur, depending on the distribution of types on the two market sides. Correspondingly, the profit of intermediary m is $\pi_m = \tau_m y^M$ if m attracts all matches, $\pi_m = \tau_m y^S_m$ if α attracts (h,ℓ) and β all other matches, or $\pi_m = 0$ if the other market attracts all matches.

Note now that for any regime intermediary β's payoff is discontinuous at $\tau^M_\beta(\tau_\alpha)$ and at $\tau^S_\beta(\tau_\alpha)$. This is also true in case $\tau^S_\beta(.) = \tau^M_\beta(.)$. Hence, a Nash equilibrium in pure strategies of the fee setting game with payoffs determined by an equilibrium matching across markets may not exist. This is confirmed by the following lemma.

Lemma 2 *If $\frac{1-\beta}{1-\alpha} \le \frac{\beta}{\alpha}\frac{y(h,h)}{y(h,\ell)}$, in the unique Nash equilibrium intermediary α chooses $\tau_\alpha = 0$, while β chooses $\tau_\beta = (\alpha-\beta)/(1-\beta)$ and all matches use β. If $\frac{1-\beta}{1-\alpha} > \frac{\beta}{\alpha}\frac{y(h,h)}{y(h,\ell)}$, a pure strategy Nash equilibrium does not exist.*

Suppose $\frac{1-\beta}{1-\alpha} > \frac{\beta}{\alpha}\frac{y(h,h)}{y(h,\ell)}$ for what follows. Existence of a Nash equilibrium in mixed strategies is ensured by the results in Dasgupta and Maskin (1986), Simon (1987). Denote the intermediaries' mixing distributions over fees τ by $F_\alpha(\tau)$ and $F_\beta(\tau)$. A fee τ_β yields intermediary β an expected profit of

$$E[\pi_\beta] = \tau_\beta Prob(\tau^M_\beta(\tau_\alpha) \le \tau_\beta \le \tau^S_\beta(\tau_\alpha))y^S_\beta + Prob(\tau_\beta < \tau^M_\beta(\tau_\alpha))y^M.$$

Likewise, when choosing τ_α platform α has expected payoff

$$E[\pi_\alpha] = \tau_\alpha Prob(\tau^M_\alpha(\tau_\beta) \le \tau_\alpha \le \tau^S_\alpha(\tau_\beta))y^S_\alpha + Prob(\tau_\alpha < \tau^M_\alpha(\tau_\beta))y^M,$$

where $\tau^M_\alpha(\tau_\beta) = (\tau^S_\beta)^{-1}(\tau_\beta)$ and $\tau^S_\alpha(\tau_\beta) = (\tau^M_\beta)^{-1}(\tau_\beta)$.

In order to characterize the mixing distributions start with the bounds on the support. Denote by $\overline{\tau}_m$ and $\underline{\tau}_m$ the upper, respectively lower bound of the support of intermediary m's mixed strategy. Bounds are defined by $F_m(\tau) = 0$ for all $\tau < \underline{\tau}_m$ and $F_m(\tau) > 0$ for $\tau > \underline{\tau}_m$, and $F_m(\tau) = 1$ for all $\tau > \overline{\tau}_m$ and $F_m(\tau) < 1$ for $\tau < \overline{\tau}_m$.

Lemma 3 *In any mixed strategy Nash equilibrium of the fee setting game the support of the intermediaries' mixing distributions satisfies (i)* $\underline{\tau}_\beta \geq \tau_\beta^M(\underline{\tau}_\alpha)$ *and* $\underline{\tau}_\alpha \geq \tau_\alpha^M(\underline{\tau}_\beta)$ *and (ii)* $\overline{\tau}_\beta \leq \tau_\beta^S(\overline{\tau}_\alpha)$ *and* $\overline{\tau}_\alpha \leq \tau_\alpha^S(\overline{\tau}_\beta)$*.*

Intermediary β's expected payoff from choosing fee τ_β is

$$E[\pi_\beta] = \tau_\beta[(1 - F_\alpha(\tau_\alpha^S(\tau_\beta)))y^M + (F_\alpha(\tau_\alpha^S(\tau_\beta)) - F_\alpha(\tau_\alpha^M(\tau_\beta)))y_\beta^S].$$

For fees τ_β it must hold that $E[\pi_\beta(\tau_\beta)] = const.$ almost everywhere on $[\underline{\tau}_\beta, \overline{\tau}_\beta]$. Otherwise a positive measure of probability could be shifted from less to more profitable fees. This in turn requires that F_α is differentiable on $(\underline{\tau}_\beta, \overline{\tau}_\beta)$ and therefore

$$\begin{aligned}(1 - F_\alpha(\tau_\alpha^S(\tau_\beta)))y^M &+ [F_\alpha(\tau_\alpha^S(\tau_\beta)) - F_\alpha(\tau_\alpha^M(\tau_\beta))]y_\beta^S \\ &= \tau_\beta \left[\frac{f_\alpha(\tau_\alpha^S(\tau_\beta))(y^M - y_\beta^S)}{\frac{\partial \tau_\beta^M(\tau)}{\partial \tau}} + \frac{f_\alpha(\tau_\alpha^M(\tau_\beta))y_\beta^S}{\frac{\partial \tau_\beta^S(\tau)}{\partial \tau}} \right]. \end{aligned} \tag{4}$$

This allows pinning down useful properties of the supports of F_α and F_β.

Lemma 4 *For any mixed strategy Nash equilibrium, the mixing distributions of intermediaries* $m, n \in \{\alpha, \beta\}$*,* $n \neq m$ *have the following properties:*

(i) There is no interval $[\tau_a, \tau_b]$ *with strictly positive probability under* $F_n(.)$*, so that both* $[\tau_m^S(\tau_a), \tau_m^S(\tau_b)]$ *and* $[\tau_m^M(\tau_a), \tau_m^M(\tau_b)]$ *have probability* 0 *under* $F_m(.)$*.*

(ii) There is no interval $[\tau_a, \tau_b]$ *with strictly positive probability and without an atom under* $F_n(.)$*, so that both* $[\tau_m^S(\tau_a), \tau_m^S(\tau_b)]$ *and* $[\tau_m^M(\tau_a), \tau_m^M(\tau_b)]$ *have strictly positive probability under* $F_m(.)$*.*

(iii) There is a unique $\tau_m^0 \in [0, 1]$*, so that if* τ_m^0 *has strictly positive probability under* $F_m(.)$*, both* $\tau_n^S(\tau_m^0)$ *and* $\tau_n^M(\tau_m^0)$ *may have strictly positive probability under* $F_n(.)$*.*

In essence, Lemma 4 states that the support of F_β has to be a projection of F_α for each point using exactly one of the mappings $\tau_\beta^S(.)$ and $\tau_\beta^M(.)$, unless there is an atom at τ_m^0 and at $\tau_n^S(\tau_m^0)$ and $\tau_n^M(\tau_n^0)$.

As a corollary to the lemma, suppose there are $\tau_a, \tau_b \in [\underline{\tau}_m, \overline{\tau}_m]$, $m \in \{\alpha; \beta\}$ such that both $\tau_n^S(\tau_a), \tau_n^M(\tau_b) \in [\underline{\tau}_n, \overline{\tau}_n]$, where $n \neq m \in \{\alpha, \beta\}$. Then, since $\tau_m^M(\tau) < \tau_m^S(\tau)$ and $(\tau_m^S)^{-1} = \tau_n^M$, it must hold that $\tau_a < \tau_b$.

Turn now to characterizing the mixing distributions in detail. To do so some more notation is required.

Definition 3 *Define* τ_α^* *by* $\tau_\alpha^* = \tau_\alpha^S(\underline{\tau}_\beta) = \tau_\alpha^M(\overline{\tau}_\beta)$ *and similarly* $\tau_\beta^* = \tau_\beta^S(\underline{\tau}_\alpha) = \tau_\beta^M(\overline{\tau}_\alpha)$.

That is, τ_m^* is a fee of intermediary m such that both the lowest and the highest fees in the other intermediary's support give exactly the threshold fees for sharing the market; this means that for any higher (lower) fee of the other intermediary m captures (loses) the entire market. The next lemma ensures the definition of τ_m^* is meaningful and details the equilibrium mixing distributions.

Lemma 5 *In any Nash equilibrium in mixed strategies of the fee setting game* τ_m^*, $m = \alpha, \beta$, *is unique. The mixing distribution* $F_m(.)$ *is of the form*

$$F_m(\tau) = \begin{cases} 1 - \frac{\kappa_m^M}{\frac{\partial \tau_m^M(\tau)}{\partial \tau} - 1 + \tau} & \text{if } \tau \leq \tau_m^* \\ \frac{y^M}{y^M - y_{-m}^S} - \frac{\kappa_m^S}{\frac{\partial \tau_m^S(\tau)}{\partial \tau} - 1 + \tau} & \text{if } \tau \geq \tau_m^*, \end{cases}$$

where $\kappa_m^S > 0$ *and* $\kappa_m^M > 0$ *are constants.*

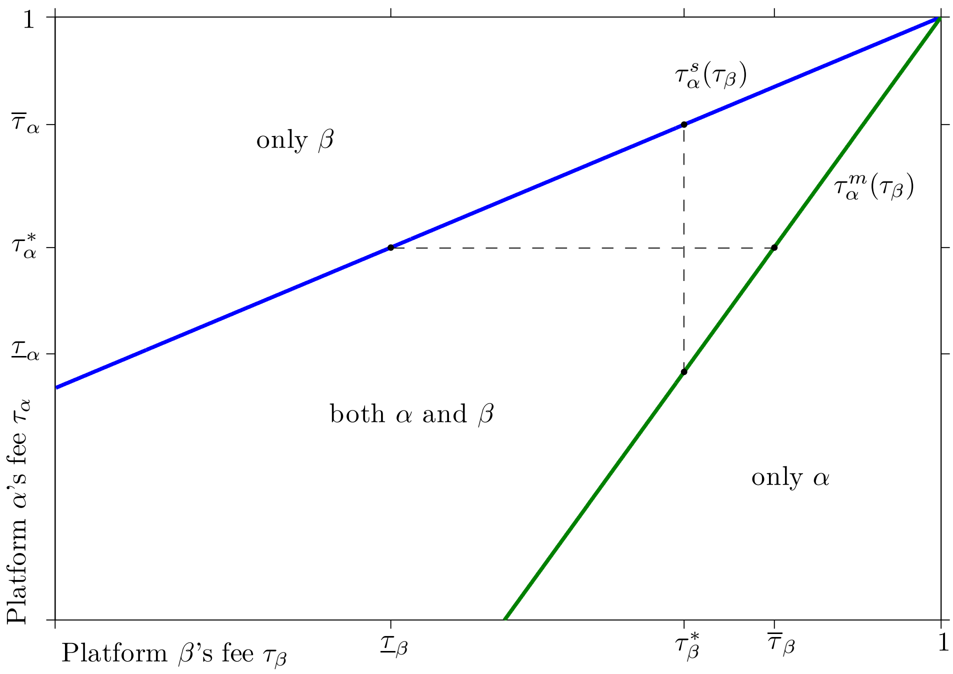

Figure 4: Support of the mixed strategies in the fee setting game.

Figure 4 sums up the results so far. The solid lines show functions τ_α^S and τ_α^M (or their inverses τ_β^M and τ_β^S). For tuples $(\tau_\alpha, \tau_\beta)$ above $\tau_\alpha^S(\tau_\beta)$

intermediary β attracts all matches, for those below $\tau_\alpha^M(\tau_\beta)$ intermediary α attracts all matches, and for those between $\tau_\alpha^S(\tau_\beta)$ and $\tau_\alpha^M(\tau_\beta)$ they share the demand. The support of the mixing distributions is characterized by τ_α^* and τ_β^*, pinning down the bounds of the supports. Moreover, the lemma implies that the mixing distribution cannot have atoms on $(\underline{\tau}_m, \overline{\tau}_m)$ except at τ_m^*, which may be compatible with a Nash equilibrium if part (iii) of Lemma 4 holds at τ_m^*.

Turn now to this exception. Suppose that distribution F_m places a point measure on τ_m^*. But then intermediary $-m$ has a profitable deviation by shifting probability mass from a neighborhood $[\underline{\tau}_{-m}, \underline{\tau}_{-m} + \epsilon)$ to a neighborhood $[\underline{\tau}_{-m} - \epsilon, \underline{\tau}_{-m})$: this generates a discrete payoff increase from y_{-m}^S to y^M in an event that has strictly positive probability (because of the point measure at τ_m^*), so that there is $\epsilon > 0$ small enough to make this deviation strictly profitable. Hence, to be consistent with a Nash equilibrium a point measure on τ_m^* implies $\underline{\tau}_{-m} = 0$. Since $\underline{\tau}_{-m} = 0$ implies that $\tau_m^* = \tau_m^S(0)$, the function F_m derived above requires that $\tau_m^* = \overline{\tau}_m$.

This is compatible with Lemma 4 if and only if $\tau_m^S(0) \geq 0$ and $\tau_m^M(0) \geq 0$ for some intermediary. As $\partial\tau_\beta^S(\tau)/\partial\tau < 1$ this may only occur for intermediary β, if $\partial\tau_\beta^M(\tau)/\partial\tau < 1$. In this case $Prob(\tau_\alpha = \overline{\tau}_\alpha) = 0$ and $Prob(\tau_\beta = \underline{\tau}_\beta) = 0$, since otherwise β has a profitable deviation by marginally decreasing $\overline{\beta}$ or marginally decreasing $\underline{\beta}$, thus obtaining y^M instead of y_β^S with strictly positive probability. This pins down the constants κ_α^S and κ_β^M. $\tau_\alpha^* = 0$ pins down $\underline{\tau}_\beta = 1 - \partial\tau_\beta^M(\tau)/\partial\tau$ and $\tau_\beta^* = \overline{\tau}_\beta = (\alpha - \beta)/(1 - \beta)$, and this in turn yields $\overline{\tau}_\alpha = 1 - (1 - \alpha)/(1 - \beta)(\partial\tau_\beta^M(\tau)/\partial\tau)^{-1}$. β's expected profit is given by $E[\pi_\beta] = \tau_\beta^* y_\beta^S > 0$, since F_α makes β indifferent between all $\tau \in [\underline{\tau}_\beta, \tau_\beta^*]$ by construction. For intermediary α expected payoff is $E[\pi_\alpha] > E[\tau_\alpha | \tau_\alpha \leq \tau_\beta^*] y_\alpha^S$. Since we know from the analysis above that both $Prob(\tau_\alpha \in (0, \tau_\alpha^S(\tau_\beta^*))) > 0$ and $Prob(\tau_\beta = \tau_\beta^*) > 0$, $E[\pi_\alpha] > 0$. This yields the first result.

Lemma 6 (Degenerate Mixed Strategy Equilibrium) *If* $\frac{\partial\tau_\beta^M(\tau)}{\partial\tau} < 1$, *a Nash equilibrium in mixed strategies is given by the mixing distributions*

$$F_\alpha(\tau) = \frac{y^M}{y^M - y_\beta^S} - \frac{y_\beta^S}{y^M} \frac{\frac{\partial\tau_\alpha^S(\tau)}{\partial\tau} - 1 + \overline{\tau}_\alpha}{\frac{\partial\tau_\alpha^S(\tau)}{\partial\tau} - 1 + \tau} \text{ and } F_\beta(\tau) = 1 - \frac{\frac{\partial\tau_\beta^M(\tau)}{\partial\tau} - 1 + \underline{\tau}_\beta}{\frac{\partial\tau_\beta^M(\tau)}{\partial\tau} - 1 + \tau}.$$

F_α has an atom at $\tau_\alpha^* = \underline{\tau}_\alpha = 0$, F_β at $\tau_\beta^* = \overline{\tau}_\beta = (\alpha - \beta)/(1 - \beta)$. Both intermediaries have strictly positive expected payoff. A separating equilibrium inducing the first best sorting outcome has strictly positive probability.

In case the distribution F_m characterized in Lemma 5 does not have an atom at τ_m^* it is easily verified that

$$\frac{\partial E[\pi_m(\tau_m)]}{\partial\tau}|_{\tau_m\leq\underline{\tau}_m} > 0 \text{ and } \frac{\partial E[\pi_m(\tau_m)]}{\partial\tau}|_{\tau_m\geq\overline{\tau}_m} < 0.$$

That is, given intermediary $-m$ mixes using F_{-m}, intermediary m cannot gain by deviating to fees on the boundary and outside the support of $F_m(.)$. Focus therefore on equilibria that have the property $\underline{\tau}_m < \tau_m^* < \overline{\tau}_m$ for $m = \alpha, \beta$ with mixing distributions given by Lemma 5, which is incompatible with a point measure on τ_i^* as argued above. Using this and the fact that $E[\pi_{-m}(\tau_{-m})] = \tau_{-m}^* y_{-m}^s$ for all $\tau_{-m} \in [\underline{\tau}_{-m}, \overline{\tau}_{-m}]$ yields

$$\frac{y^M}{y^M - y^S_{-m}} - \frac{\kappa_m^S}{\frac{\partial\tau_m^S(\tau)}{\partial\tau} - (1-\tau_m^*)} = \frac{\frac{\overline{\tau}_{-m}}{\underline{\tau}_{-m}} - \frac{y^M}{y^S_{-m}}}{1 + \frac{\overline{\tau}_{-m}}{\underline{\tau}_{-m}} - \frac{y^M}{y^S_{-i}}} = 1 - \frac{\kappa_m^M}{\frac{\partial\tau_m^M(\tau)}{\partial\tau} - (1-\tau_m^*)}. \quad (5)$$

This pins down κ_m^S and κ_m^M as functions of τ_m^*, since $\underline{\tau}_{-m} = \tau_{-m}^M(\tau_m^*)$ and $\overline{\tau}_{-m} = \tau_{-m}^S(\tau_m^*)$. Noting that, if for instance $\tau_m \geq \tau_m^*$,

$$\tau_m^* y_m^S = E[\pi_m] = \tau_m y_m^s (1 - F_{-m}(\tau_{-m}^M(\tau_m))),$$

and using the expressions for κ_m^S and κ_m^M as functions of τ_m^* we have that

$$\tau_m^* \left(\frac{y_m^S}{y^M - y_m^S} \frac{1}{\underline{\tau}_m} - \frac{1}{\overline{\tau}_m} \right) = \frac{y_m^S}{y^M - y_m^S}. \quad (6)$$

Since $\underline{\tau}_m$ and $\overline{\tau}_m$ are determined by τ_{-m}^* we have two equations and two unknowns. Straightforward but tedious calculations reveal that τ_α^* and τ_β^* have the following properties.

Lemma 7 *Properties of τ_α^* and τ_β^*:*

(i) For $m = \alpha, \beta$

$$\frac{\partial\tau_m^*(\tau_{-m}^*)}{\partial\tau_{-m}} > 0 \textit{ and } \frac{\partial^2\tau_m^*(\tau_{-m}^*)}{\partial^2\tau_{-m}^*} > 0$$

for all τ_{-m} such that $\tau_m^*(\tau_{-m}^*) \in [0, 1]$.

(ii) There is $\overline{\tau}_\beta^* \in (0, 1)$ such that $\lim_{\tau_\beta^*\to\overline{\tau}_\beta^*} \tau_\alpha^*(\tau_\beta^*) = \infty$, and $\tau_\beta(1) > 1$.

(iii) $\tau_\alpha^*(\underline{\tau}_\beta^*) = 0$ implies $0 < \underline{\tau}_\beta^* < \overline{\tau}_\beta^*$.

(iv) If $\tau_\beta^*(0) > 0$ then $\tau_\beta^*(0) < \underline{\tau}_\beta^*$ if $\tau_\beta^M(0)y^M < \tau_\beta^S(0)y_\beta^S$ and $\tau_\beta^*(0) = \underline{\tau}_\beta^*$ if $\tau_\beta^M(0)y^M > \tau_\beta^S(0)y_\beta^S$.

(v) τ_α^* *increases in* α, τ_β^* *decreases in* β.

Part (i) implies that $\tau_\alpha^*(\tau)$ and $(\tau_\beta^*)^{-1}(\tau)$ cross exactly twice. The remaining parts ensure that they cross exactly once in the unit square. Part (ii) implies that any intersection has to occur for $\tau_\alpha < 1$ and $\tau_\beta < \underline{\tau}_\beta^*$. Part (iii) and (iv) ensure that the first intersection requires $\tau_\alpha < 0$ when $\tau_\beta^M(0)y^M < \tau_\beta^S(0)y_\beta^S$, that is whenever choosing a fee τ_β to share the market is not payoff dominated by monopolizing the market by choosing $\tau_\beta^M(0)$. If $\tau_\beta^M(0)y^M \geq \tau_\beta^S(0)y_\beta^S$ there emerges the degenerate mixed strategy equilibrium described in Lemma 6. This implies the following statement.

Lemma 8 (Nondegenerate Mixed Strategy Equilibrium) *Given* α *and* β *there is a unique tuple* $(\tau_\alpha^*, \tau_\beta^*)$ *such that*

$$\tau_m^* \left(\frac{y_m^S}{y^M - y_m^S} \frac{1}{\underline{\tau}_m(\tau_{-m}^*)} - \frac{1}{\overline{\tau}_m(\tau_{-m}^*)} \right) = \frac{y_M^S}{y^M - y_M^S} \text{ for } m = \alpha, \beta.$$

$(\tau_\alpha^*, \tau_\beta)$ *fully determines the mixing distributions, given in Lemma 5 where* κ_m^S *and* κ_m^M *are functions of* τ_m^* *given by (5), of a Nash equilibrium in mixed strategies with* $\underline{\tau}_m < \tau_m^* < \overline{\tau}_m$*. In equilibrium intermediaries have payoff* $E[\pi_\alpha] = \tau_\alpha^* y_\alpha^S > 0$ *and* $E[\pi_\beta] = \tau_\beta^* y_\beta^S > 0$*. If* $\tau_\beta^M(0)y^M < \tau_\beta^S(0)y_\beta^S$*, it is the only Nash equilibrium; otherwise there is another Nash equilibrium, which is of the form described in Lemma 6 and Pareto dominated for intermediaries.*

Applying the implicit function theorem to the system of two equations (6) yields

$$\frac{\partial \tau_\alpha^*}{\partial \alpha} > 0 \text{ for all } \alpha > \frac{\frac{\beta}{1-\beta}\frac{y(h,h)}{y(h,\ell)}}{1 + \frac{\beta}{1-\beta}\frac{y(h,h)}{y(h,\ell)}},$$

independent of whether $\frac{\partial \tau_\beta^M(\tau)}{\partial \tau} = \frac{1-\alpha}{1-\beta}\frac{y(h,\ell)}{y(\ell,\ell)}$ or $\frac{\partial \tau_\beta^M(\tau)}{\partial \tau} = \frac{\alpha}{\beta}\frac{y(h,\ell)}{y(h,h)}$. Similarly,

$$\frac{\partial \tau_\beta^*}{\partial \beta} < 0 \text{ for all } \beta < \left(1 + \frac{1-\alpha}{\alpha}\frac{y(h,h)}{y(h,\ell)}\right)^{-1}.$$

If the conditions on α and β in the conditions above do not hold, Lemma 2 applies and intermediary β captures the whole market. Hence, expected payoffs within all equilibrium regimes, described by Lemmata 2, 6, and 8, increase in τ_α (decrease in τ_β).

This implies that $\beta = \epsilon$. Suppose otherwise. Then β must yield higher payoff for intermediary β than ϵ. Since β's payoff strictly decreases in β in all mixed strategy equilibria, $\beta = \alpha\frac{y(h,h)-y(h,\ell)}{y(h,h)}$, which maximizes β's payoff in a pure strategy equilibrium, see Lemma 2. In a pure strategy equilibrium market α has payoff $\pi_\alpha = 0$. Independently of whether first or second mover chose β, there is a profitable deviation for α or β: if $s_1 = \beta$, clearly any $s_2 < \beta$ yields higher payoff to the second mover than $s_2 = \alpha$. If $s_2 = \beta$, clearly $s_1 = \epsilon$ yields higher payoff to the first mover than $s_1 = \alpha$. For both first and second mover, a best reply to $\beta = \epsilon$ is $\alpha = 1 - \epsilon$. The first mover will choose $s_1 = \epsilon$ or $s_1 = 1 - \epsilon$, whichever yields higher payoff.

The weight of realizations $\tau < \tau_m^*$ and $\tau > \tau_m^*$ in the equilibrium mixing distributions increases in the slope of $\tau_{-m}^M(\tau)$ and $\tau_{-m}^S(\tau)$, respectively. As ϵ decreases, the slope of $\tau_m^M(\tau)$ increases, while the one of $\tau_m^S(\tau)$ decreases. Therefore mixing distributions put more and more probability mass on $\tau < \tau_m^*$ as ϵ decreases. As the slopes approach infinity and 0, respectively, the weight that the joint probability distribution of $(\tau_\alpha, \tau_\beta)$, given by the mixing distributions, puts on $\{(\tau_\alpha, \tau_\beta) : \underline{\tau}_\alpha \leq \tau_\alpha < \tau_\alpha^*, \underline{\tau}_\beta < \tau_\beta \tau_\beta^*\}$ approaches 1. For all these realizations sorting across markets takes place, with (h, ℓ) matches using α and (ℓ, h) matches using β. This is summarized in the following proposition.

Proposition 7 (Competing Intermediaries) *Equilibrium choices of sharing rules satisfy $\alpha = 1 - \epsilon$ and $\beta = \epsilon$. The first mover has higher payoff. As ϵ approaches 0 the probability of sorting across markets as an outcome approaches 1.*

That is, competition of intermediaries can ensure a framework of contractual and other market institutions determining attainable payoffs to parties in business relationship that allows to mitigate coordination problems due to non-transferabilities, which may arise for instance when asymmetric information, renegotiation, or behavioral norms are pertinent. Moreover, since in the limit, as ϵ approaches 0, the intermediaries approach full surplus extraction. This suggests that also a single intermediary has an incentive to offer different contractual institutions to induce sorting across markets. This may not always be feasible, for instance when attainable payoffs reflect the reputation of the intermediary to be buyer or seller friendly.

4 Conclusion

In assignment problems, that is when economic outcomes of interest depend on the characteristics of agents who interact with each other, a market equilibrium allocation does not necessarily induce a surplus efficient allocation when utility is not perfectly transferable. This paper proposed a mechanism to restore surplus efficiency in two-sided markets when there are limits to compensation within matches: sorting across markets. Moreover, sufficient institutional diversity of market places to ensure such sorting across markets may be the result of Bertrand competition between market places. Thus allocations that are characterized by matches heterogenous in type, which are likely precluded by limits to compensation, may arise endogenously when this is surplus efficient.

A further concern arises when characteristics of agents that matches are based on are the result of agents' choices. An example for this might be choice of education acquisition by workers and of technology by firms before matching on a labor market. The "correct" sorting to be induced by a desirable mechanism has then to ensure that investment incentives generated by equilibrium payoffs are efficient (see e.g. Gall et al., 2009). Although Cole et al. (2001) find that in general a sharing of surplus exists that supports an equilibrium with surplus efficient investments, this does not suffice to guarantee that sorting across markets using these payoffs implies surplus efficient investments. For this to hold also deviations from equilibrium levels have to be rewarded by the marginal social product, which requires a minimum level of utility transferability for payoffs on each market. Characterizing conditions such that efficiency of investments is ensured under sorting across markets is an important task for future research.

A Mathematical Appendix

Proof of Proposition 1

Start by deriving the utility possibility frontier in match (a,b) explicitly. With the limited liability constraint $w \leq 1$

$$u^b(u^a) = (q+\sqrt{2q-1})\begin{cases} \frac{r(a,b)}{4q} & \text{if } u^a \leq \frac{(1-q)^2}{4q^2}\frac{r(a,b)}{2} \\ (1-q)r(a,b)/2 & \text{if } u^a \geq (1-q)^2\frac{r(a,b)}{2} \\ \frac{1}{1-q}\left(\sqrt{2r(a,b)u^a} - \frac{2qu^a}{1-q}\right) & \text{otherwise.} \end{cases}$$

$$u^a(u^b) = (1-q)^2\begin{cases} \frac{r(a,b)}{2} & \text{if } u^b \leq (q+\sqrt{2q-1})(1-q)r(a,b) \\ \frac{r(a,b)}{8q^2} & \text{if } u^b \geq (q+\sqrt{2q-1})\frac{r(a,b)}{4q} \\ \frac{r(a,b)\left(1+\sqrt{1-\frac{4qu^b}{(q+\sqrt{2q-1})r(a,b)}}\right)^2}{8q^2} & \text{otherwise.} \end{cases}$$

$u^b(u^a)$ can be parameterized by the match (a,b) they are subject to writing $u^b(u^a,a,b)$ and $u^a(u^b,a,b)$. Then for $u \in [0,(q+\sqrt{2q-1})r(\ell,\ell)/(4q)]$

$$u^a(a,b,u) = (1-q)^2\begin{cases} \frac{r(a,b)}{2} & \text{if } u \leq (q+\sqrt{2q-1})(1-q)r(a,b) \\ \frac{r(a,b)\left(1+\sqrt{1-\frac{4qu}{(q+\sqrt{2q-1})r(a,b)}}\right)^2}{8q^2} & \text{otherwise.} \end{cases}$$

That is, $u^a(a,b,u) \leq (1-q)^2 r(a,b)/2$. Therefore and

$$u^b(a,b,u^a(.)) = (.)\begin{cases} \frac{r(a,b)}{4q} & \text{if } u^a(.) \leq \frac{(1-q)^2}{4q^2}\frac{r(a,b)}{2} \\ \frac{1}{1-q}\left(\sqrt{2r(a,b)u^a(.)} - \frac{2qu^a(.)}{1-q}\right) & \text{otherwise.} \end{cases}$$

Note that $u^b(a,b,u^a(.))$ strictly decreases in $u^a(.)$ for $u^a(.) > \frac{(1-q)^2}{4q^2}\frac{r(a,b)}{2}$.

Generalized increasing differences holds if

$$u^b(h,h,u^a(h,\ell,u)) > u^b(\ell,h,u^a(\ell,\ell,u)) \tag{GID}$$

for all $u \in [0,(q+\sqrt{2q-1})r(\ell,\ell)/(4q)]$. Since $u^b(a,b,u)$ decreases in u a sufficient condition is

$$\sqrt{r(h,h)r(h,\ell)} - qr(h,\ell) > \frac{r(h,\ell)}{4q}.$$

Note that this condition is necessary if $r(h,\ell)/r(\ell,\ell) \geq 2-4q(1-q)$, since then $u^a(\ell,\ell,u) \leq (1-q)^2 r(h,\ell)/(8q^2)$ for some $u \in [0,(q+\sqrt{2q-1})r(\ell,\ell)/(4q)]$. The sufficient condition yields

$$\sqrt{\frac{r(h,h)}{r(h,\ell)}} > q + \frac{1}{4q}.$$

Hence, $\sqrt{\frac{r(h,h)}{r(h,\ell)}} > 5/4$ implies condition (GID) for all $q \in [1/2, 1]$.

Applying Proposition 1 from Legros and Newman (2007) the matching equilibrium is positive assortative meaning that the type of an agent's match is increasing in the agent's type. That is, the all possible (h, h) matches are exhausted, the remaining h agents match with ℓ agents, and all remaining possible (ℓ, ℓ) matches are exhausted.

Proof of Proposition 2

First we show necessity. That is, suppose an assignment and payoffs $(u_i)_{i \in I}$ are an equilibrium with intermediaries $\{\phi_{m_1}, \phi_{m_2}, ..., \phi_{m_n}\}$. Note first that payoffs (u_i, u_j) for any match of $i \in A$ and $j \in B$ are feasible with respect to ϕ_m, since $(u_i, u_j) \in \phi_{c_i}(a_i, b_j) \subseteq \phi_m(a_i, b_j)$. Feasibility of payoffs of unmatched agents follows by definition. It remains to show stability. Since the assignment and payoffs are an equilibrium there does not exist $m' \in \{m_1, m_2, ..., m_n\}$ and $i \in A$ and $j \in B$ such that $u'_i > u_i$ and $u'_j > u_j$ for some $(u'_i, u'_j) \in \phi_{m'}(a_i, b_j)$. This implies there do not exist $i \in A$ and $j \in B$ such that $u'_i > u_i$ and $u'_j > u_j$ for some $(u'_i, u'_j) \in \bigcup_{m=m_1,m_2,...,m_n} \phi_m(a_i, b_i)$.

Suppose an assignment and payoffs $(u_i)_{i \in I}$ are an equilibrium allocation with an intermediary with attainable payoffs ϕ. Again start by verifying feasibility. For any match of $i \in A$ and $j \in B$ payoffs are feasible, $(u_i, u_j) \in \phi$. Since $\phi = \bigcup_{m=m_1,m_2,...,m_n} \phi_m$ this implies there is some intermediary $m \in m_1, m_2, ..., m_n$ such that $(u_i, u_j) \in \phi_m$. Assigning choice $c_i = c_j = m$ then ensures feasibility of the payoffs also for an allocation with many intermediaries. Turn now to stability. The assignment and payoffs $(u_i)_{i \in I}$ are an equilibrium with respect to payoffs ϕ. Therefore there is no pair $i \in A$ and $j \in B$ such that $u'_i > u_i$ and $u'_j > u_j$ for some $(u'_i, u'_j) \in \phi$. Since $\phi(a_i, b_i) = \bigcup_{m=m_1,m_2,...,m_n} \phi_m(a_i, b_i)$ this implies neither is there $m \in \{m_1, m_2, ..., m_n\}$ with some $(u'_i, u'_j) \in \phi_m$ such that $u'_i > u_i$ and $u'_j > u_j$. This implies in particular that there is no $(u'_i, u'_j) \in \phi_{c_i}$ such that $u'_i > u_i$ and $u'_j > u_j$. Combined with feasibility of payoffs above this establishes that assignment, payoffs $(u_i)_{i \in I}$ and choices $(c_i)_{i \in I}$ generate a equilibrium in each market place $m_1, m_2, ..., m_n$, and that there is no pair $i \in A$ and $j \in B$ that has a mutually strictly positive gain by deviating across markets. This completes the proof. $\square$

Proof of Proposition 4

Since entrepreneurs are scarce, the market rents go to the financiers. If only European contracts are available, financiers obtain $r(\ell,\ell)/4$ in (ℓ,ℓ) and $r(\ell,h)/4$ in (ℓ,h) matches because of the entrepreneurs' limited liability. This means h entrepreneurs need to give ℓ principals payoff $r(\ell,b)/4$ in (h,b) matches, which leaves them with

$$r(h,b)(1+\sqrt{1-r(\ell,b)/r(h,b)})^2/8.$$

h type entrepreneurs prefer (h,h) matches giving the principal $r(\ell,h)/4$ to (h,ℓ) matches giving the principal $r(\ell,\ell)/4$, which means the equilibrium matching will be positive assortative, if

$$\sqrt{r(h,h)}+\sqrt{r(h,h)-r(h,\ell)}>\sqrt{r(h,\ell)}+\sqrt{r(h,\ell)-r(\ell,\ell)}. \tag{7}$$

Under U.S. style contracts ℓ entrepreneurs compete for financiers and obtain $r(\ell,b)(1-q)^2/(8q^2)$. Because entrepreneurs are borrowing constrained, financiers obtain $(q+\sqrt{2q-1})r(\ell,b)/(4q)$. Hence, an h type entrepreneur in a match (h,b) obtains

$$r(h,b)(1+\sqrt{1-r(\ell,b)/r(h,b)})^2\frac{(1-q)^2}{8q^2}.$$

Comparing payoffs, an h entrepreneur finds an (h,h) match preferable to an (h,ℓ) match, giving the principal payoff $(q+\sqrt{2q-1})r(\ell,b)/(4q)$, if condition (7) holds.

If both contracts are available the equilibrium match will exhaust all (h,ℓ) and (ℓ,h) matches as shown above. this must be the case since the set of attainable payoff without limited liability contains attainable with limited liability; therefore if sorting across market can be supported by the set of attainable payoffs with limited liability, it can also be supported by the bigger set of attainable payoffs without limited liability. As capital is scarce, all matched ℓ type entrepreneurs will use U.S. style contracts, since they give higher payoff to financiers, as will all h type financiers.

Proof of Proposition 5

Suppose without loss of generality that (h,ℓ) matches use intermediary m and (ℓ,h) use m'. Then there are payoffs $(u_a,u_b)\in\phi(h,\ell)$ such that

$$\begin{aligned} &u_a\geq\max\{\underline{u}_a(h);u'_a:(u'_a,u'_b)\in\phi'(h,b),b=\ell,h\} \text{ and}\\ &u_b\geq\max\{\underline{u}_b(\ell);u'_b:(u'_a,u'_b)\in\phi'(a,\ell),a=\ell,h\} \end{aligned} \tag{8}$$

and $(u'_a, u'_b) \in \phi_{m'}(h, \ell)$ with $u'_a = \max\{u_a : (u_a, u'_b) \in \phi'(h, \ell)\}$ and $u'_b = \max\{u_b : (u'_a, u_b) \in \phi'(h, \ell)\}$, i.e. (u'_a, u'_b) is on the Pareto frontier, such that

$$\begin{aligned} u'_a &\geq \max\{\underline{u}_a(\ell); u_a : (u_a, u_b) \in \phi(\ell, b), b = \ell, h\} \text{ and} \\ u'_b &\geq \max\{\underline{u}_b(h); u_b : (u_a, u_b) \in \phi(a, h), a = \ell, h\} \end{aligned} \tag{9}$$

Set $\alpha = u'_a/(u'_a + u'_b)$, and $y(a, b) = \max_{(u_a,u_b)\in\phi'(a,b): \frac{u_a}{u_a+u_b}=\alpha} u_a + u_b$ for $a, b = \ell, h$. Let $\phi_\alpha(a, b) = \{(u_a, u_b) : u_a \leq \alpha y(a, b), u_b \leq (1 - \alpha)y(a, b)$, then in an equilibrium with intermediaries m and α there must be sorting across markets as well, since by definition both sets of conditions (8) and (9) are satisfied.

Proof of Proposition 6

Consider an equilibrium with intermediaries α and β. Examine first the possibility of (h, ℓ) matches using β and (ℓ, h) matches using α. An h type from A and an ℓ type from B are strictly better off by forming a (h, ℓ) match using β if

$$\beta y(h, \ell) \geq \alpha y(h, h) \text{ and } (1 - \beta)y(h, \ell) \geq (1 - \alpha)y(\ell, \ell),$$

where the latter only needs to hold if there are unmatched ℓ agents from A, i.e. in case $\mu > 1 - \mu$. Agents in (ℓ, h) matches using α prefer this if

$$\alpha y(\ell, h) \geq \beta y(\ell, \ell) \text{ and } (1 - \alpha)y(\ell, h) \geq (1 - \beta)y(h, h),$$

where the former only needs to hold if $\mu < 1-\mu$. Summing up the conditions on β yields

$$\begin{aligned} \beta &\geq \max\left\{\alpha\frac{y(h, h)}{y(h, \ell)}; \alpha\frac{y(h, \ell)}{y(h, h)} + \frac{y(h, h) - y(h, \ell)}{y(h, h)}\right\} \text{ and} \\ \beta &\leq \min\left\{\alpha\frac{y(\ell, \ell)}{y(h, \ell)} + \frac{y(h, \ell) - y(\ell, \ell)}{y(h, \ell)}; \alpha\frac{y(h, \ell)}{y(\ell, \ell)}\right\}. \end{aligned} \tag{10}$$

Depending on which conditions in (10) bind three different cases arise. The first case arises if $y(\ell, \ell)/(y(h, \ell) + y(\ell, \ell)) \leq \alpha \leq y(h, \ell)/(y(h, \ell) + y(h, h))$, yielding the condition

$$1 - (1 - \alpha)\frac{y(h, \ell)}{y(h, h)} \leq \beta \leq 1 - (1 - \alpha)\frac{y(\ell, \ell)}{y(h, \ell)},$$

which can always be satisfied by some $\beta \in [0,1]$ because of property (DD). The second case of (10) is

$$\alpha\frac{y(h,\ell)}{y(h,h)} + \frac{y(h,h)-y(h,\ell)}{y(h,h)} \leq \beta \leq \alpha\frac{y(h,\ell)}{y(\ell,\ell)}, \tag{11}$$

if $\alpha \leq y(\ell,\ell)/(y(h,\ell)+y(\ell,\ell))$. Finally, (10) may become

$$\alpha\frac{y(h,h)}{y(h,\ell)} \leq \beta \leq \alpha\frac{y(\ell,\ell)}{y(h,\ell)} + \frac{y(h,\ell)-y(\ell,\ell)}{y(h,\ell)}, \tag{12}$$

if $y(h,\ell)/(y(h,\ell)+y(h,h)) \leq \alpha$. This means the binding constraints on α for existence of some $\beta \in [0,1]$ to ensure (10) are (11) and (12), yielding

$$\frac{y(\ell,\ell)}{y(h,\ell)}\frac{y(h,h)-y(h,\ell)}{y(h,h)-y(\ell,\ell)} \leq \alpha \leq \frac{y(h,\ell)-y(\ell,\ell)}{y(h,h)-y(\ell,\ell)}. \tag{13}$$

If $\mu > 1-\mu$ the lower bound is not a necessary condition for sorting across markets, if $\mu < 1-\mu$ the upper bound is not necessary.

Suppose now that (h,ℓ) matches use α and (ℓ,h) matches β. An h type from B and an ℓ type from A are strictly prefer to match using β if

$$\beta y(\ell,h) > \alpha y(\ell,\ell) \text{ and } (1-\beta)y(\ell,h) > (1-\alpha)y(h,h),$$

where the former only needs to hold if $\mu < 1-\mu$. Agents in (h,ℓ) matches using α prefer this if

$$\alpha y(h,\ell) > \beta y(h,h) \text{ and } (1-\alpha)y(h,\ell) > (1-\beta)y(\ell,\ell),$$

where the latter is necessary only if $\mu > 1-\mu$. Calculations analogous to the ones above yield a condition on α for existence of some $\beta \in [0,1]$ to ensure that the above conditions hold:

$$\frac{y(h,h)-y(h,\ell)}{y(h,h)-y(\ell,\ell)} \leq \alpha \leq \frac{y(h,h)}{y(h,\ell)}\frac{y(h,\ell)-y(\ell,\ell)}{y(h,h)-y(\ell,\ell)}. \tag{14}$$

Again, if $\mu > 1-\mu$ the lower bound is not a necessary condition for sorting across markets, if $\mu < 1-\mu$ the upper bound is not necessary.

Combining conditions (13) and (14) yields the condition in the proposition. If it is satisfied there is $\beta \in [0,1]$ so that in an equilibrium with intermediaries α and β (equivalent to an equilibrium with attainable payoffs $\phi_\alpha(a,b) \cup \phi_\beta(a,b)$ for $a,b = \ell,h$) all possible pairs (ℓ,h) and (h,ℓ) form, and all remaining possible homogeneous (h,h) or (ℓ,ℓ) matches. Under property (DD) this maximizes aggregate surplus.

Proof of Lemma 1

Demand for intermediaries depending on sharing rules α, β and prices τ_α, τ_β is determined as follows. Note first that for $(1-\tau_\beta)\frac{1-\beta}{1-\alpha} > 1-\tau_\alpha > (1-\tau_\beta)\frac{\beta}{\alpha}$ all agents from A prefer α for the same match, and all agents from B prefer β. Since agents from B are scarce, all homogeneous matches will form in β. Therefore, for α to capture the entire market

$$(1-\alpha)(1-\tau_\alpha) > (1-\beta)(1-\tau_\beta). \tag{15}$$

Moreover, at least one the following has to hold to ensure that either h types from B or ℓ types from A do not prefer to match using β:

$$\beta(1-\tau_\beta)y(h,\ell) < \alpha(1-\tau_\alpha)y(\ell,\ell) \text{ or } (1-\beta)(1-\tau_\beta)y(h,\ell) < (1-\alpha)(1-\tau_\alpha)y(h,h).$$

Note that (15) implies $(1-\beta)(1-\tau_\beta)y(h,\ell) < (1-\alpha)(1-\tau_\alpha)y(h,h)$. Finally, at least one of the following has to hold, so that either h types from A or ℓ types from B do not prefer to match using β:

$$\beta(1-\tau_\beta)y(h,\ell) < \alpha(1-\tau_\alpha)y(h,h) \text{ or } (1-\beta)(1-\tau_\beta)y(h,\ell) < (1-\alpha)(1-\tau_\alpha)y(\ell,\ell),$$

Note here that (15) implies $\beta(1-\tau_\beta)y(h,\ell) < \alpha(1-\tau_\alpha)y(h,h)$, since $\alpha > \beta$. This establishes the first part of the lemma.

For β to capture the entire market necessarily $(1-\alpha)(1-\tau_\alpha) < (1-\beta)(1-\tau_\beta)$ has to hold. To ensure that either h types from B or ℓ types from A do not prefer to match using α, one of the following has to hold:

$$\beta(1-\tau_\beta)y(\ell,\ell) > \alpha(1-\tau_\alpha)y(h,\ell) \text{ or } (1-\beta)(1-\tau_\beta)y(h,h) > (1-\alpha)(1-\tau_\alpha)y(h,\ell).$$

The necessary condition implies the second inequality. To ensure that either h types from A or ℓ types from B do not prefer to match using α, one of the following has to hold:

$$\beta(1-\tau_\beta)y(h,h) > \alpha(1-\tau_\alpha)y(h,\ell) \text{ or } (1-\beta)(1-\tau_\beta)y(\ell,\ell) > (1-\alpha)(1-\tau_\alpha)y(h,\ell),$$

Either of these inequalities or the necessary condition may bind, so that $\tau_\beta^M(\tau_\alpha) = \tau_\beta^S(\tau_\alpha)$ if $\frac{1-\beta}{1-\alpha} \leq \frac{\beta}{\alpha}\frac{y(h,h)}{y(h,\ell)}$, $\tau_\beta^M(\tau_\alpha) = 1-(1-\tau_\alpha)\frac{1-\alpha}{1-\beta}\frac{y(h,\ell)}{y(\ell,\ell)}$ if $\frac{\beta}{\alpha}\frac{y(h,h)}{y(h,\ell)} < \frac{1-\beta}{1-\alpha} < \frac{\beta}{\alpha}\frac{y(h,h)}{y(\ell,\ell)}$, and $\tau_\beta^M(\tau_\alpha) = 1-(1-\tau_\alpha)\frac{\alpha}{\beta}\frac{y(h,\ell)}{y(h,h)}$ if $\frac{1-\beta}{1-\alpha} > \frac{\beta}{\alpha}\frac{y(h,h)}{y(\ell,\ell)}$. This establishes the second part of the lemma.

Suppose now that $\tau_\beta^M(\tau_\alpha) \leq \tau_\beta \leq \tau_\beta^S(\tau_\alpha)$. If $\tau_\beta^S(\tau_\alpha) = \tau_\beta^M(\tau_\alpha)$, β captures the entire market. Suppose otherwise. All h types from A and ℓ types from

B prefer to match using β. Moreover, h types from B prefer (h, h) or (h, ℓ) matches using β to (h, ℓ) matches using α. ℓ types from B are indifferent between (ℓ, ℓ) matches using either intermediary.

If $\tau_\beta^S(\tau_\alpha) > \tau_\beta^M(\tau_\alpha)$ then h types from A and ℓ types from B prefer to match using α to segregating in (ℓ, ℓ) $((h, h))$ matches using β, and at least one type of agent does strictly so. Since market side B is scarce and $(1 - \tau_\alpha)(1 - \alpha) < (1 - \tau_\beta)(1 - \beta)$, all homogeneous matches use β. For the same reason and since h types from B prefer β, (ℓ, h) matches use β.

Proof of Lemma 2

Turn to the case $\tau_\beta^M(\tau_\alpha) = \tau_\beta^S(\tau_\alpha)$ first. That is, $\frac{1-\beta}{1-\alpha} \leq \frac{\beta}{\alpha} \frac{y(h,h)}{y(h,\ell)}$. Intermediary β's payoff given τ_α is

$$\pi_\beta = \begin{cases} \tau_\beta y^M & \text{if } \tau_\beta \leq \tau_\beta^M(\tau_\alpha) \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, intermediary α's payoff given τ_β is

$$\pi_\alpha = \begin{cases} \tau_\alpha y^M & \text{if } \tau_\beta > \tau_\beta^M(\tau_\alpha) \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, both $\tau_\beta^M(\tau_\alpha) > 0$ and $\tau_\alpha > 0$ are not consistent with an equilibrium as either intermediary would find it profitable undercut the other one. Since $\tau_\beta^M(0) > 0$, for $\tau_\beta = \tau_\beta^M(0)$ intermediary β captures the entire market, while α cannot undercut β, and β has no profitable deviation: decreasing τ_β diminishes monopoly profits, and increasing τ_β loses the entire demand to α. Therefore $\tau_\alpha = 0$ and $\tau_\beta = (\alpha - \beta)/(1 - \beta)$ is the unique pure strategy Nash equilibrium of the price setting game whenever $\frac{1-\beta}{1-\alpha} \leq \frac{\beta}{\alpha} \frac{y(h,h)}{y(h,\ell)}$.

Suppose now the contrary, which results in $\tau_\beta^M(\tau_\alpha) < \tau_\beta^S(\tau_\alpha)$. Then intermediary β's payoff given τ_α is

$$\pi_\beta = \begin{cases} \tau_\beta y^M & \text{if } \tau_\beta < \tau_\beta^M(\tau_\alpha) \\ \tau_\beta y_\beta^S & \text{if } \tau_\beta^M(\tau_\alpha) \leq \tau_\beta \leq \tau_\beta^S(\tau_\alpha) \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, platform α's payoff given τ_β is

$$\pi_\alpha = \begin{cases} \tau_\alpha y^M & \text{if } \tau_\beta > \tau_\beta^S(\tau_\alpha) \\ \tau_\alpha y_\alpha^S & \text{if } \tau_\beta^S(\tau_\alpha) \geq \tau_\beta \geq \tau_\beta^M(\tau_\alpha) \\ 0 & \text{otherwise.} \end{cases}$$

Hence, a best reply for β to some τ_α is either $\tau_\beta^S(\tau_\alpha)$ (this is strictly positive as $\tau_\beta^S(0) > 0$), in case $\tau_\beta^S(\tau_\alpha) y_\beta^S \geq \tau_\beta^M(\tau_\alpha)$, or it is not defined as β would like to choose τ_β as close as possible to, but strictly smaller than $\tau_\beta^M(\tau_\alpha)$. Similarly, a best reply for α to some τ_β is $\max\{0; (\tau_\beta^M)^{-1}(\tau_\beta)\}$ or, if $(\tau_\beta^M)^{-1}(\tau_\beta) y_\alpha^s < (\tau_\beta^S)^{-1}(\tau_\beta) y^m$ it is not defined.

Hence, any pure strategy equilibrium τ_α, τ_β has to satisfy $\tau_\beta = \tau_\beta^S(\tau_\alpha)$ and $\tau_\alpha = (\tau_\beta^M)^{-1}(\tau_\beta)$. Note that $\tau_\beta^S(\tau_\alpha) \neq \tau_\beta^M(\tau_\alpha)$ for $\tau_\alpha \neq 1$, however. For $\tau_\alpha = 1 = \tau_\beta$ a marginal decrease in the fee of either intermediary will capture the entire market yielding a discrete increase of profit. Therefore the price setting game has no pure strategy Nash equilibrium when $\frac{1-\beta}{1-\alpha} > \frac{\beta}{\alpha} \frac{y(h,h)}{y(h,\ell)}$.

Proof of Lemma 3

Suppose that $\underline{\tau}_\beta < \tau_\beta^M(\underline{\tau}_\alpha)$. Then there is $\underline{\tau}'_\beta \in (\underline{\tau}_\beta; \tau_\beta^M(\underline{\tau}_\alpha))$ with $F_\beta(\underline{\tau}'_\beta) > 0$. Hence, choosing F'_β with $F'_\beta(\tau) = 0$ for all $\tau_\beta < \underline{\tau}'_\beta$ and $F'_\beta(\tau_\beta) = F_\beta(\tau_\beta)$ for all $\tau_\beta \geq \underline{\tau}'_\beta$ strictly increases expected payoff, since the probabilities of the events of capturing the entire market, sharing the market, and losing the market are the same under F_β and F'_β. Similarly, intermediary α has a strictly profitable deviation of shifting probability mass to $(\tau_\alpha^M(\underline{\tau}_\beta), \tau_\alpha^S(\overline{\tau}_\beta))$.

Suppose now that $\overline{\tau}_\beta > \tau_\beta^S(\overline{\tau}_\alpha)$. Then there is $\overline{\tau}'_\beta \in (\tau_\beta^S(\underline{\tau}_\alpha); \underline{\tau}_\beta)$ with $F_\beta(\overline{\tau}'_\beta) < 1$. Hence, choosing $F'_\beta(\tau)$ such that $F'_\beta(\tau) > F_\beta(\tau)$ for $\tau_\beta^M(\underline{\tau}_\beta) < \tau < \tau_\beta^S(\overline{\tau}_\beta)$ and $F'_\beta(\tau) = F_\beta(\tau) + (1 - F_\beta(\overline{\tau}'_\beta))$ for $\tau_\beta^S(\underline{\tau}_\beta) < \tau < \overline{\tau}'_\beta$, strictly increases payoff, since for all $\tau_\beta > \tau_\beta^S(\overline{\tau}_\alpha)$ intermediary β's payoff is 0 with certainty, while it is strictly positive for $\tau_\beta^M(\underline{\tau}_\beta) < \tau_\beta < \tau_\beta^S(\overline{\tau}_\beta)$. Again, an analogous argument holds for intermediary α.

Proof of Lemma 4

For part (i) suppose that $[\tau_a, \tau_b]$, $\tau_b \leq \overline{\tau}_\beta$, has probability $\epsilon > 0$ under F_β. Suppose that $[\tau_\alpha^S(\tau_a), \tau_\alpha^S(\tau_b)]$ and $[\tau_\alpha^M(\tau_a), \tau_\alpha^M(\tau_b)]$ have probability 0 under $F_\alpha(.)$. Then shifting probability mass ϵ to the point τ_b strictly increases β's expected payoff as revenue increases while the probabilities $Prob(\tau_\beta < \tau_\beta^M(\tau_\alpha)) \geq 0$ and $Prob(\tau_\beta \leq \tau_\beta^S(\tau_\alpha)) > 0$ remain unaffected. $Prob(\tau_\beta \leq \tau_\beta^S(\tau_\alpha)) > 0$ since otherwise β makes zero expected profit and could choose $\tau_\beta = \tau_\beta^S(\underline{\tau_\alpha})$. A similar argument applies to intermediary α and $F_\alpha(.)$, assuming strictly positive expected profit, which implies $Prob(\tau_\alpha \leq \tau_\alpha^S(\tau_\beta)) > 0$.

For the remaining parts suppose again that $[\tau_a, \tau_b]$, $\tau_b \leq \overline{\tau}_\beta$, has strictly positive probability under F_β. Let both $[\tau^S_\alpha(\tau_a), \tau^S_\alpha(\tau_b)]$ and $[\tau^M_\alpha(\tau_a), \tau^M_\alpha(\tau_b)]$ have strictly positive probability under $F_\alpha(.)$. Then $E[\pi_\alpha] = const.$ on both $[\tau^S_\alpha(\tau_a), \tau^S_\alpha(\tau_b)]$ and $[\tau^M_\alpha(\tau_a), \tau^M_\alpha(\tau_b)]$ implies that F_β is differentiable and satisfies both

$$\frac{y^M}{y^M - y^S_\alpha} - F_\beta(\tau^S_\beta(\tau_\alpha))) = \tau_\alpha \frac{f_\beta(\tau^S_\beta(\tau_\alpha))}{\frac{\partial \tau^M_\alpha(\tau)}{\partial \tau}}.$$

for $\tau_\alpha \in [\tau^M_\alpha(\tau_a), \tau^M_\alpha(\tau_b)]$ and

$$(1 - F_\beta(\tau^M_\beta(\tau_\alpha))) = \tau_\alpha \frac{f_\beta(\tau^M_\beta(\tau_\alpha))}{\frac{\partial \tau^S_\alpha(\tau)}{\partial \tau}}$$

for $\tau_\alpha \in [\tau^S_\alpha(\tau_a), \tau^S_\alpha(\tau_b)]$. That is,

$$\frac{y^M}{y^M - y^S_\alpha} - F_\beta(\tau) = f_\beta(\tau)\left[\frac{\partial \tau^S_\beta(\tau)}{\partial \tau} - 1 + \tau\right] \text{ and } 1 - F_\beta(\tau) = f_\beta(\tau)\left[\frac{\partial \tau^M_\beta(\tau)}{\partial \tau} - 1 + \tau\right]$$

for all $\tau \in [\tau_a, \tau_b]$. Differentiating both sides once more with respect to τ yields either $f_\beta = 0$ or a contradiction to Lemma 1, implying $\frac{\partial \tau^M_\beta(\tau)}{\partial \tau} \neq \frac{\partial \tau^S_\beta(\tau)}{\partial \tau}$ for $\frac{1-\beta}{1-\alpha} > \frac{\beta}{\alpha}\frac{y(h,h)}{y(h,\ell)}$ as assumed. Hence, $E[\pi_\alpha] = const.$ on both $[\tau^S_\alpha(\tau_a), \tau^S_\alpha(\tau_b)]$ and $[\tau^M_\alpha(\tau_a), \tau^M_\alpha(\tau_b)]$ is only consistent with $F_\beta(.)$ having an atom $\tau^*_\beta \in [\tau_a, \tau_b]$, with $\tau^S_\alpha(\tau^*_\beta) y^S_\alpha = \tau^M_\alpha(\tau^*_\beta) y^M$. This defines a unique τ^*_β. An analogous arguments holds for F_α yielding τ^*_α defined by $\tau^S_\beta(\tau^*_\alpha) y^S_\beta = \tau^M_\beta(\tau^*_\alpha) y^M$.

Proof of Lemma 5

The distribution F_α has to ensure that $E[\pi_\beta] = const.$, requiring either

$$y^M - F_\alpha(\tau^S_\alpha(\tau))(y^M - y^S_\beta) = \tau f_\alpha(\tau^S_\alpha(\tau))(y^M - y^S_\beta)\frac{\partial \tau^S_\alpha(\tau)}{\partial \tau}, \qquad (16)$$

for all $\tau \in [\underline{\tau}_\beta, \overline{\tau}_\beta]$ with $\tau^S_\alpha(\tau) \in [\underline{\tau}_\alpha, \overline{\tau}_\alpha]$ or

$$1 - F_\alpha(\tau^M_\alpha(\tau)) = \tau f_\alpha(\tau^M_\alpha(\tau))\frac{\partial \tau^M_\alpha(\tau)}{\partial \tau}, \qquad (17)$$

for all $\tau \in [\underline{\tau}_\beta, \overline{\tau}_\beta]$ with $\tau^M_\alpha(\tau) \in [\underline{\tau}_\alpha, \overline{\tau}_\alpha]$. Differentiating (16) and solving the equation, noting that $\tau^S_\alpha(\tau) = 1 - (1-\tau)\frac{\partial \tau^S_\alpha(\tau)}{\partial \tau}$, yields

$$F_\alpha(\tau) = \frac{y^M}{y^M - y^S_\beta} - \frac{\kappa^S_\alpha}{\frac{\partial \tau^S_\alpha(\tau)}{\partial \tau} - 1 + \tau},$$

with $\kappa^S_\alpha > 0$, for all $\tau \in [\underline{\tau}_\alpha, \overline{\tau}_\alpha]$ with $\tau = \tau^S_\alpha(\tau_\beta)$, $\tau_\beta \in [\underline{\tau}_\beta, \overline{\tau}_\beta]$. Similarly, differentiating (17) and solving the resulting equation yields

$$F_\alpha(\tau) = 1 - \frac{\kappa^M_\alpha}{\frac{\partial \tau^M_\alpha(\tau)}{\partial \tau} - 1 + \tau}$$

with $\kappa_\alpha^M > 0$, for all $\tau \in [\underline{\tau}_\alpha, \overline{\tau}_\alpha]$ with $\tau = \tau_\alpha^M(\tau_\beta)$, $\tau_\beta \in [\underline{\tau}_\beta, \overline{\tau}_\beta]$.

Repeating this exercise for $E[\pi_\alpha] = const.$ yields

$$F_\beta(\tau) = \frac{y^M}{y^M - y_\alpha^S} - \frac{\kappa_\beta^S}{\frac{\partial \tau_\beta^S(\tau)}{\partial \tau} - 1 + \tau},$$

with $\kappa_\beta^S > 0$, for all $\tau \in [\underline{\tau}_\beta, \overline{\tau}_\beta]$ with $\tau = \tau_\beta^S(\tau_\alpha)$, $\tau_\alpha \in [\underline{\tau}_\alpha, \overline{\tau}_\alpha]$ and

$$F_\beta(\tau) = 1 - \frac{\kappa_\beta^M}{\frac{\partial \tau_\beta^M(\tau)}{\partial \tau} - 1 + \tau},$$

with $\kappa_\beta^M > 0$, for all $\tau \in [\underline{\tau}_\beta, \overline{\tau}_\beta]$ with $\tau = \tau_\beta^M(\tau_\alpha)$, $\tau_\alpha \in [\underline{\tau}_\alpha, \overline{\tau}_\alpha]$.

Establish now existence of τ_m^*. That is, there is $\tau \in [\underline{\tau}_m, \overline{\tau}_m]$ such that both $\underline{\tau}_{-m} \leq \tau_{-m}^M(\tau) < \tau_{-m}^S(\tau) \leq \overline{\tau}_{-m}$ for $m = \alpha, \beta$. Suppose the contrary. Then for some m there is no $\tau \in [\underline{\tau}_{-m}, \overline{\tau}_{-m}]$ such that $\tau_m^S(\tau) \in [\underline{\tau}_m, \overline{\tau}_m]$. But this implies that $F_m(\tau) = 1 - \frac{\kappa_\beta^M}{\frac{\partial \tau_\beta^M(\tau)}{\partial \tau} - 1 - \overline{\tau}_m} < 1$, which in turn requires that F_m has an atom at $\overline{\tau}_m$. But this cannot be part of a Nash equilibrium in mixed strategy as intermediary m can strictly increase payoff by shifting probability mass from $\overline{\tau}_m$ to $\tau_m^S(\underline{\tau}_{-m}) > \overline{\tau}_m$, which increases revenue from sharing the market but does not alter the probability of this event given $[\underline{\tau}_{-m}, \overline{\tau}_{-m}]$.

By parts (i) and (ii) of Lemma 4 τ_m^* must be unique, since otherwise necessarily there would be intervals in the support of F_m such that both their projection using mapping τ_{-m}^S and τ_{-m}^M would be in the support of F_{-m}.

Therefore, and since $\tau_m^S(.)$ and $\tau_m^M(.)$ are strictly increasing functions, for all $\underline{\tau}_m \leq \tau \leq \tau_m^*$ necessarily $\tau_{-m}^S(\tau) \in [\underline{\tau}_{-m}, \overline{\tau}_{-m}]$ but $\tau_{-m}^S(\tau) \notin [\underline{\tau}_{-m}, \overline{\tau}_{-m}]$. For all $\tau_m^* \leq \tau \leq \overline{\tau}_m$ necessarily $\tau_{-m}^M(\tau) \in [\underline{\tau}_{-m}, \overline{\tau}_{-m}]$ but $\tau_{-m}^S(\tau) \notin [\underline{\tau}_{-m}, \overline{\tau}_{-m}]$. This concludes the characterization of $F_\alpha(\tau)$ and $F_\beta(\tau)$.

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